



# RESEARCH LABORATORY OF NONLINEAR UNDERACTUATED SYSTEMS

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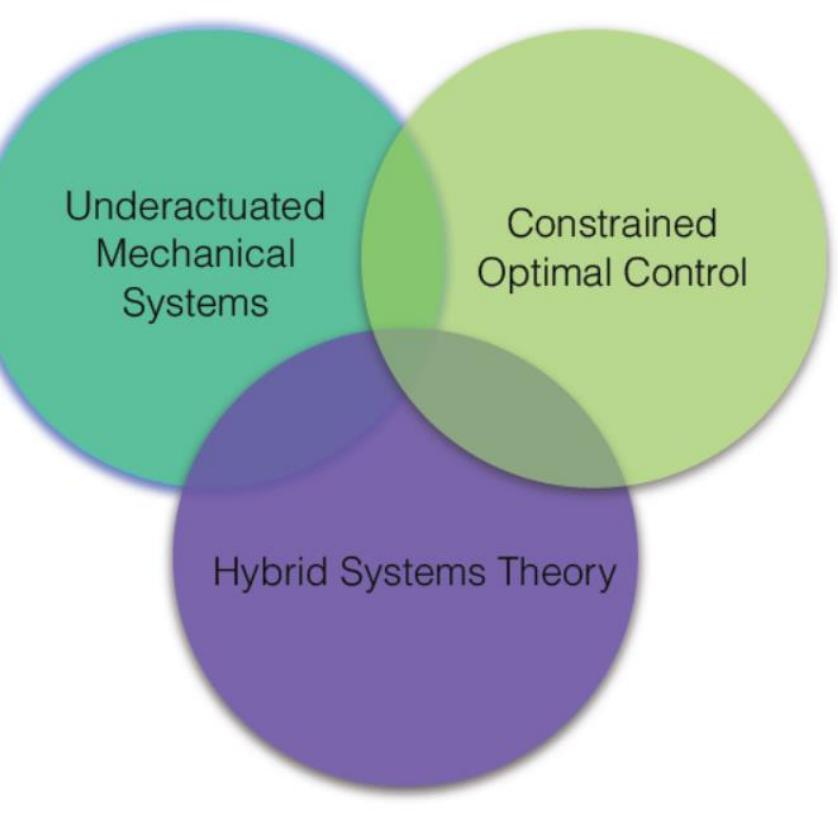


## OBJECTIVES

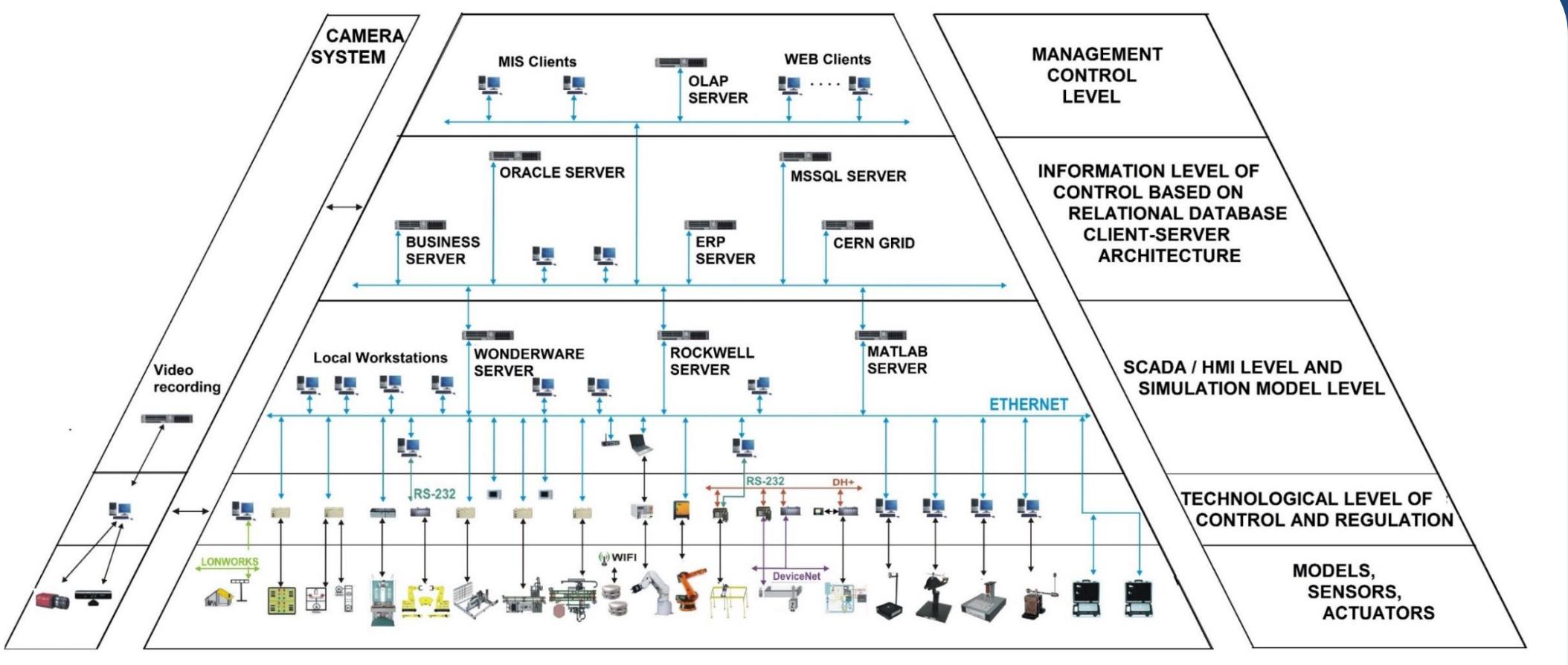
**Underactuated systems**, defined as mechanical systems with fewer control inputs than degrees of freedom, appear in a broad range of applications including aerospace, marine and locomotive systems. The motivation behind the research into underactuation is the ability to control nonlinear systems without complete control authority by exploiting their natural dynamics. This is similar to how biological systems execute motions involving a loss of instantaneous control authority. Underactuated devices are therefore expected to be more efficient, simpler and more reliable than their fully actuated alternatives. However, control of underactuated devices is more complex to design theoretically. Our laboratory is the first research group to comprehensively deal with the topic of underactuated systems in Slovakia. Worldwide research groups which have inspired our research include:



The general objective of our research group is to identify and tackle the open research problems occurring at the mutual overlaps between three principal areas: **mathematical modeling** of underactuated mechanical systems, **optimal control** of nonlinear systems subject to constraints, and **hybrid systems theory**. We have often been employing the concept of cyber-physical systems.



**Cyber-Physical Systems** (CPS) integrate the dynamics of the physical processes with those of the software and networking, providing abstractions and modeling, design, and analysis techniques for the integrated whole. Physical subsystems in CPS operate in a time continuum, whereas cyber subsystems are composed of discrete, step-by-step operations. A key CPS challenge is to conjoin the engineering abstractions for continuous dynamics (such as differential equations) with computer science abstractions (such as algorithms).



In order to contribute to the modeling and control education at the DCAI-FEI TU, our results are being integrated into the research and teaching activities of the Center of Modern Control Techniques and Industrial Informatics at the DCAI: <http://kyb.fei.tuke.sk>. The results of hybrid modeling and control design for the considered **cyber-physical systems** are implemented at all levels of the **distributed control system infrastructure** in accordance with the **Industry 4.0** strategy.

## TEAM & TEACHING

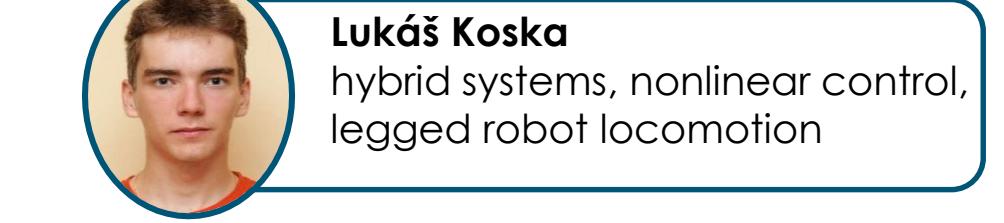
### TEAMLEADER



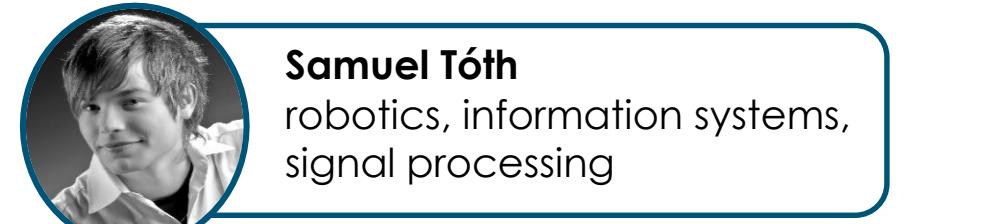
Slávka JADLOVSKÁ, PhD.

- mathematical modeling, constrained optimal control, nonlinear control, hybrid systems, mechatronics and robotics
- underactuated systems (benchmarks, manipulators, hybrid control, robot locomotion – biped gait)
- 26 published scientific works in this area (10 in indexed journals/proceedings)
- 52 citations (20 in indexed journals)
- 2011-2017 – supervisor/consultant to 18 bachelor and 14 diploma theses

### PHD STUDENTS



Lukáš Koska  
hybrid systems, nonlinear control, legged robot locomotion



Samuel Tóth  
robotics, information systems, signal processing

### MASTER STUDENTS



### CURRENT COURSES (FIELD OF STUDY: CYBERNETICS)

**Introduction to Control Engineering** – linear system theory, elementary mathematical modeling, PID controller design, introduction to cyber-physical systems

**Embedded Systems** – development of applications based on microcontrollers, programming in machine-oriented languages, principles of Internet of Things

**Simulation Systems** – introduction to technical computing using MATLAB/Simulink (modeling and simulation of linear/nonlinear systems, feedback control)

**Computer Systems in Control** – development of PC applications with the focus on standard interfaces

**Optimal Control of Hybrid Systems** – nonlinear system theory, linearization, optimal control design, stability, elementary hybrid system theory

**Control and Artificial Intelligence** – advanced controller design (predictive/adaptive control), experimental identification, intelligent control

**Distributed Control Systems** – complex control in manufacturing organizations (PLC, SCADA, information systems), principles of Industry 4.0

**Management Information Systems** – multidimensional analysis of business data, introduction to big data processing

### PAST COURSES

**Computers and Algorithms** – algorithm design and C programming, introduction to computing and hardware

**Simulation Systems in Business Informatics** – MATLAB programming with business applications (optimization)

**Control of Technological Processes** – principles of PLC control

**Protocols and Interfaces** – introduction to computer networks and the Internet

**Introduction to Nonlinear Systems** – nonlinear system theory (linearization, stability, nonlinear control design)

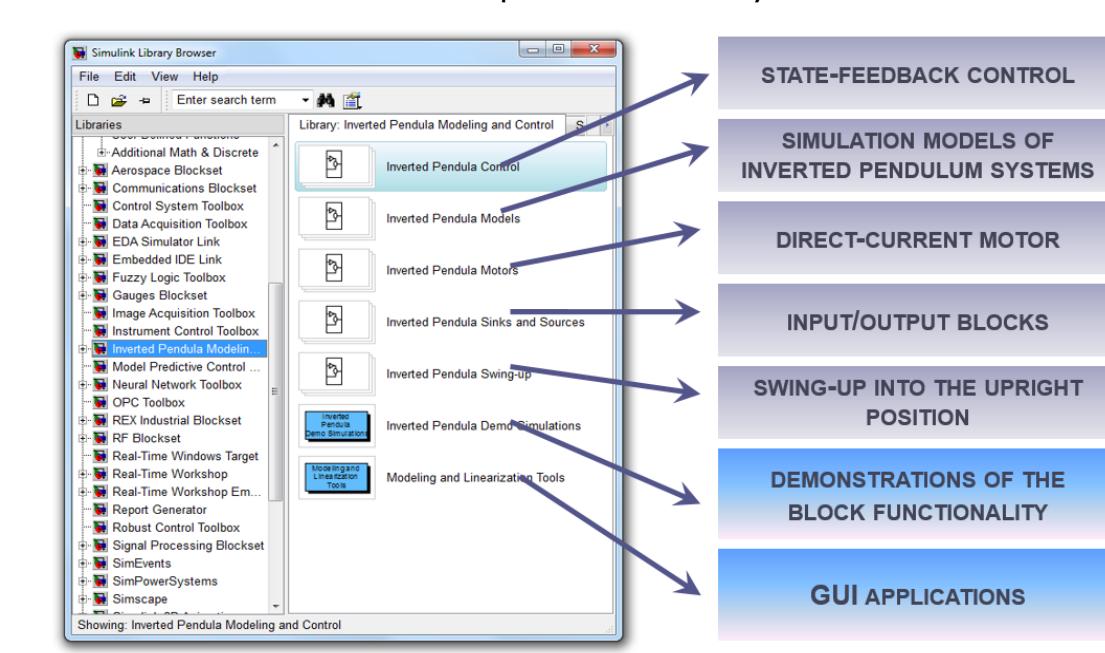
**Models and Control of Industrial Processes** – modeling of mechanical/mechatronic/electrical/hydraulic/pneumatic systems, experimental identification, control design

## OUTLINE, CONTRIBUTION AND PERSPECTIVES

### MATHEMATICAL MODELING OF UNDERACTUATED SYSTEMS USING LAGRANGIAN MECHANICS

Benchmark underactuated systems, such as inverted pendulum systems, two-link planar robots – Acrobot and Pendubot, the Inertia Wheel Pendulum or the conveyer-crane system create complex low-order nonlinear dynamics which enables us to gain insight into the principles of modeling and control of advanced, high-order underactuated systems. We have developed of a set of algorithms which determine the Lagrange equations of motion for a selected benchmark underactuated system. We have specifically introduced the concept of a generalized (n-link) inverted pendulum system, which allows us to treat an arbitrary system of interconnected inverted pendula as a particular instance of the system of pendula attached to a given stabilizing base, such as a cart (in 2D/3D) or a rotary arm.

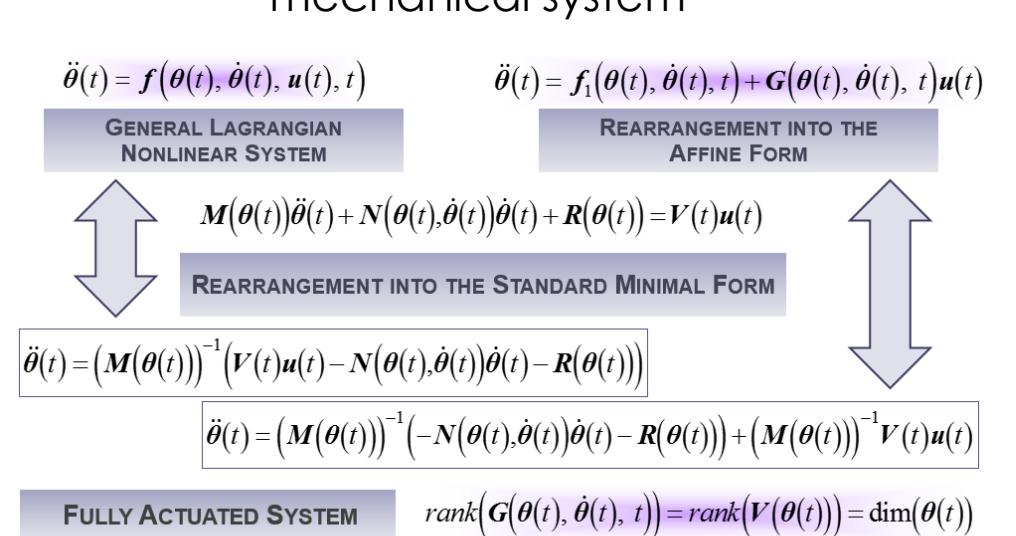
**Inverted Pendula Modeling and Control (IPMAC)** – Simulink block library for modeling and control of inverted pendulum systems



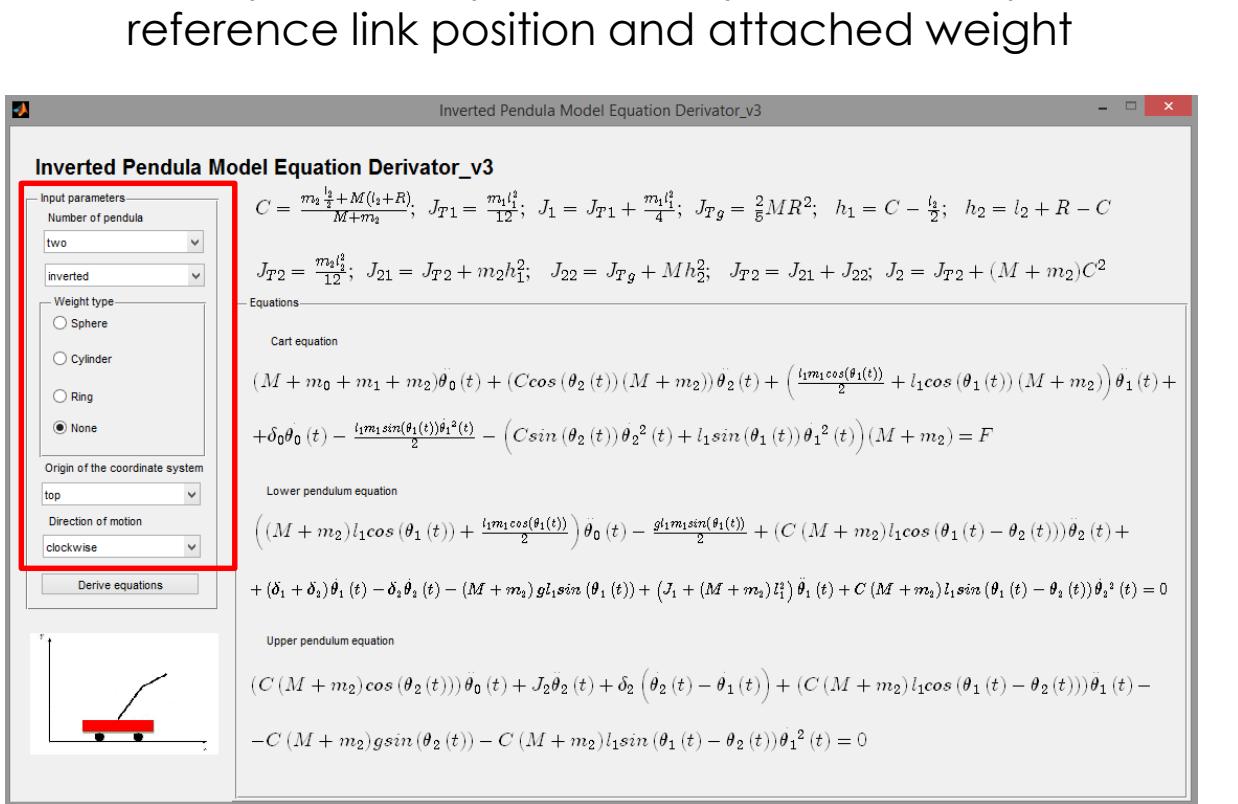
Our ultimate goal was to complete a readily available collection of mathematical and simulation models of underactuated mechanical systems serving as a testbed model basis in simulation experiments exploring their typical properties (such as analyses of open-loop dynamics and phase portraits) and testing linear and nonlinear control strategies.

The results obtained so far have provided a starting point for our current research of advanced underactuated systems, such as legged robots and the mechanism of robot walking, wheeled inverted pendulum systems or unmanned aerial vehicles.

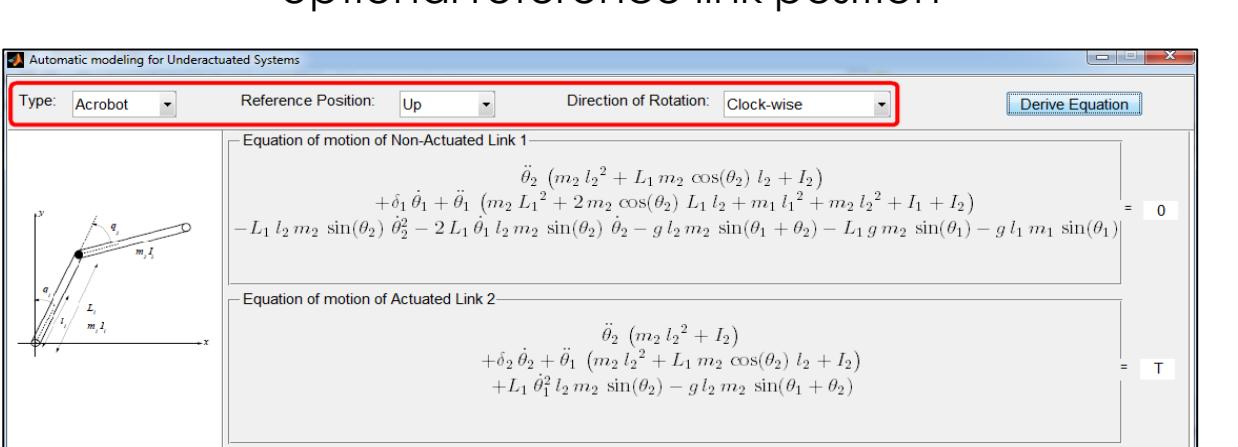
Application of Euler-Lagrange equations in modeling of a multi-body underactuated mechanical system



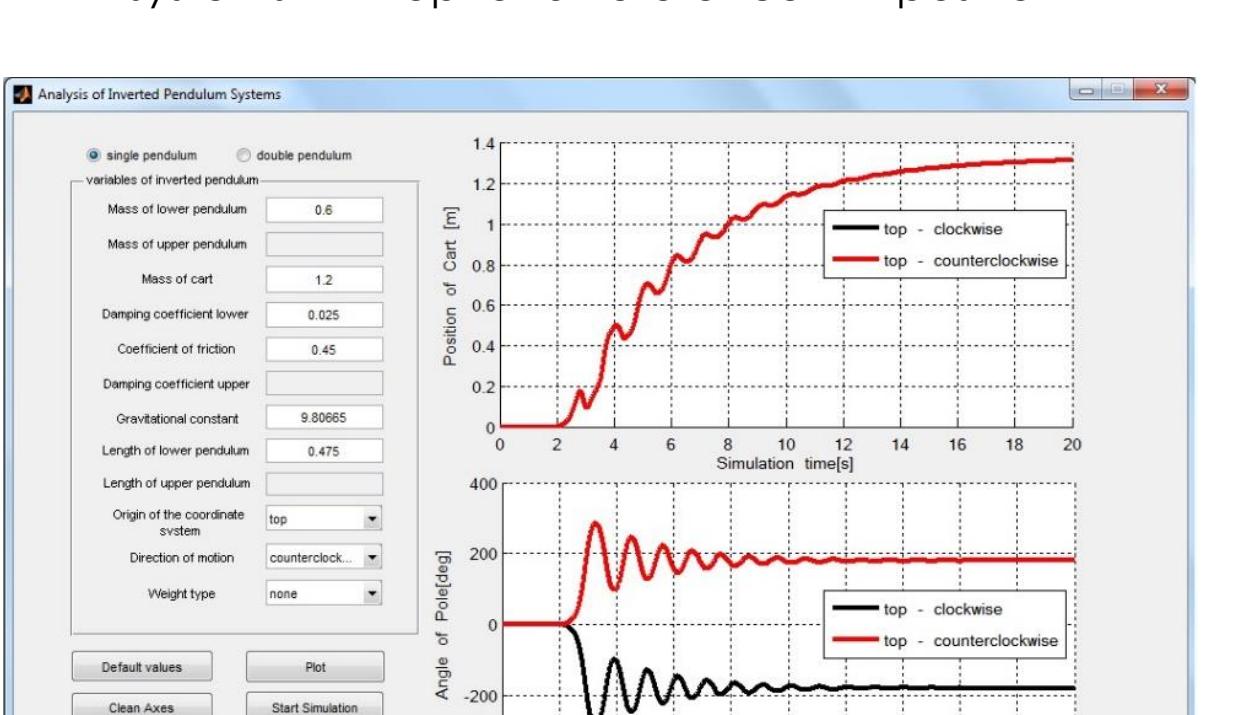
Automatic generation of motion equations for classical and rotary inverted pendulum systems with optional reference link position and attached weight



Automatic generation of motion equations for an Acrobot, Pendubot and inertia Wheel Pendulum with optional reference link position



Open-loop analysis of classical inverted pendulum systems with optional reference link position



### OPTIMAL CONTROL OF UNDERACTUATED SYSTEMS

Fully actuated systems possess a number of strong structural properties (feedback linearizability, passivity, linear parametrizability) which facilitate controller design. These are usually lost in underactuated systems. At the same time, undesirable properties (higher relative degree, nonminimum phase behavior) emerge.

The goal of optimal control design for a linear, time-invariant system is to determine such feedback control so that a given optimality criterion is achieved. In case the considered linear system is actually a linear approximation of a nonlinear system around a given equilibrium, optimal techniques for linear systems yield an approximate, locally near-optimal stabilizing solution with guaranteed closed-loop stability and robustness.

State space representation and linear approximation of underactuated systems

$$\dot{x}(t) = (\theta(t) \quad \dot{\theta}(t))^T = (\ddot{x}_1(t) \quad \ddot{x}_2(t))^T$$

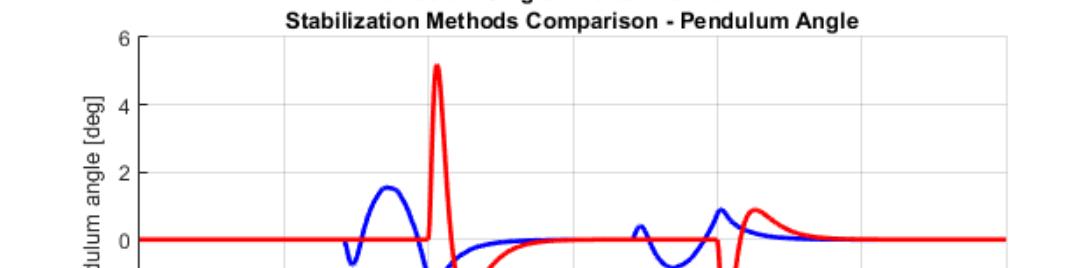
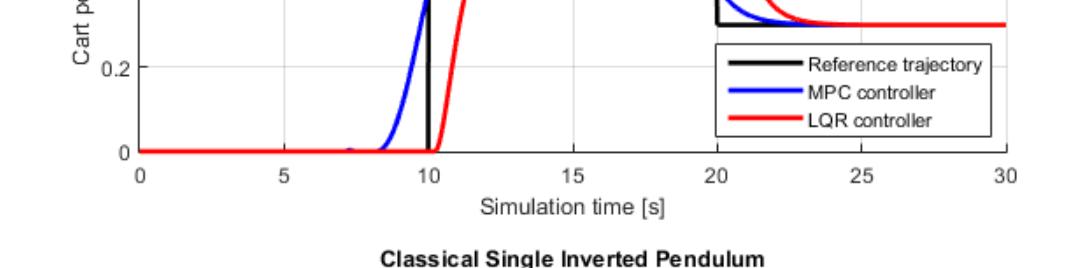
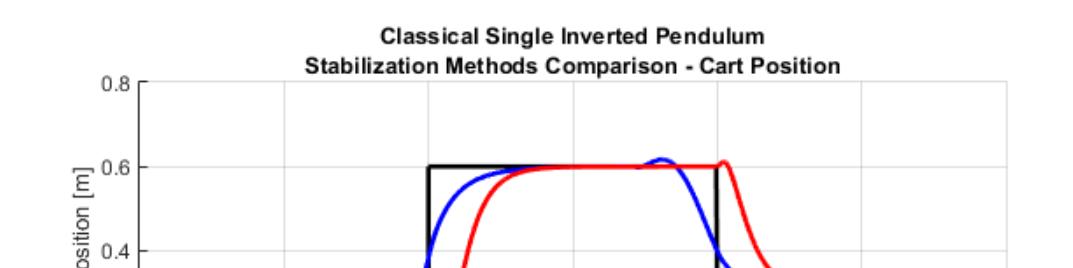
$$y(t) = g(x(t), \dot{x}(t)) = Cx(t) + Bu(t)$$

$$\dot{x}(t) = f(x(t), u(t)) = \begin{bmatrix} 0 & I \\ -M^{-1}B & 0 \end{bmatrix}x(t) + \begin{bmatrix} 0 \\ -M^{-1}C \end{bmatrix}u(t)$$

$$\ddot{x}(t) = Ax(t) + Bu(t)$$

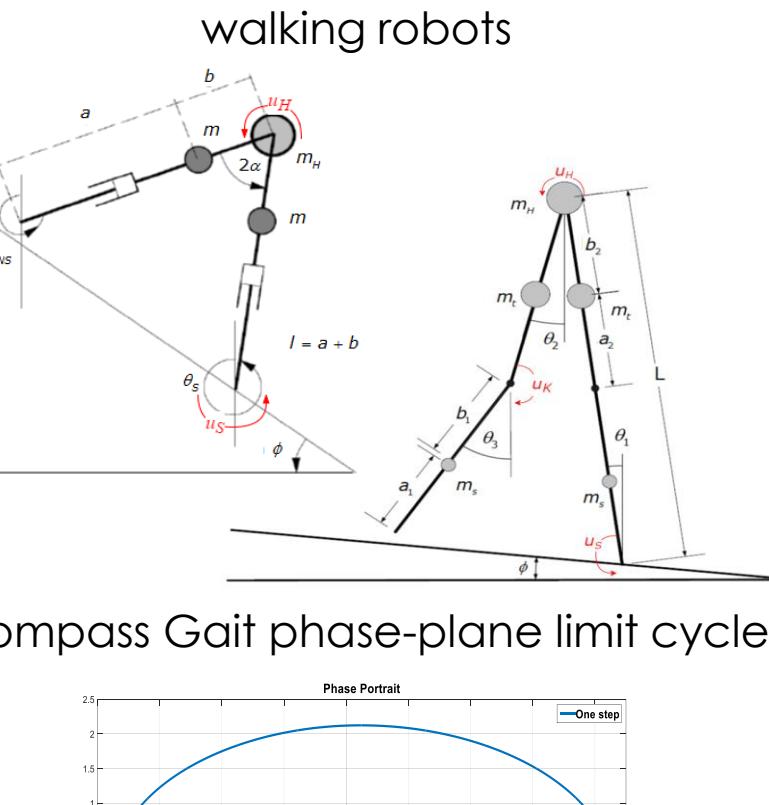
We have implemented and verified LQ control algorithms in a variety of control structures, and evaluated the need for nonlinear control techniques such as the state-dependent Riccati equation.

**Model predictive control (MPC)** is a discrete-time optimal control technique in which the control action for each time step is computed by solving an online optimization problem in finite time while considering input/state constraints. To solve problems arising from the structure of underactuated system, suitable adjustment of the MPC algorithm is required.

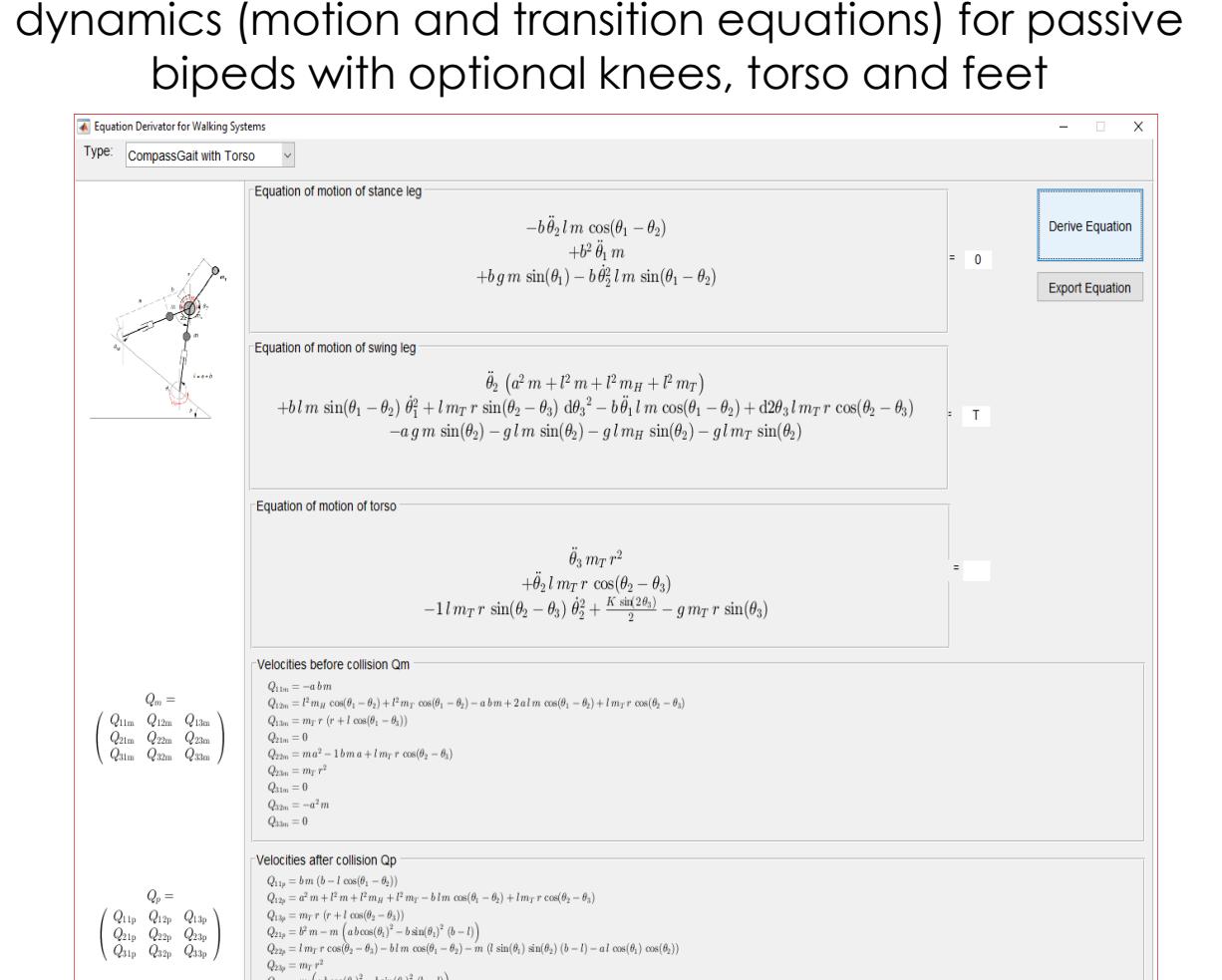


### HYBRID SYSTEMS THEORY FOR MODELING AND CONTROL OF UNDERACTUATED SYSTEMS

Examples of benchmark passive walking robots



Automatic generation of continuous and discrete dynamics (motion and transition equations) for passive bipeds with optional knees, torso and feet

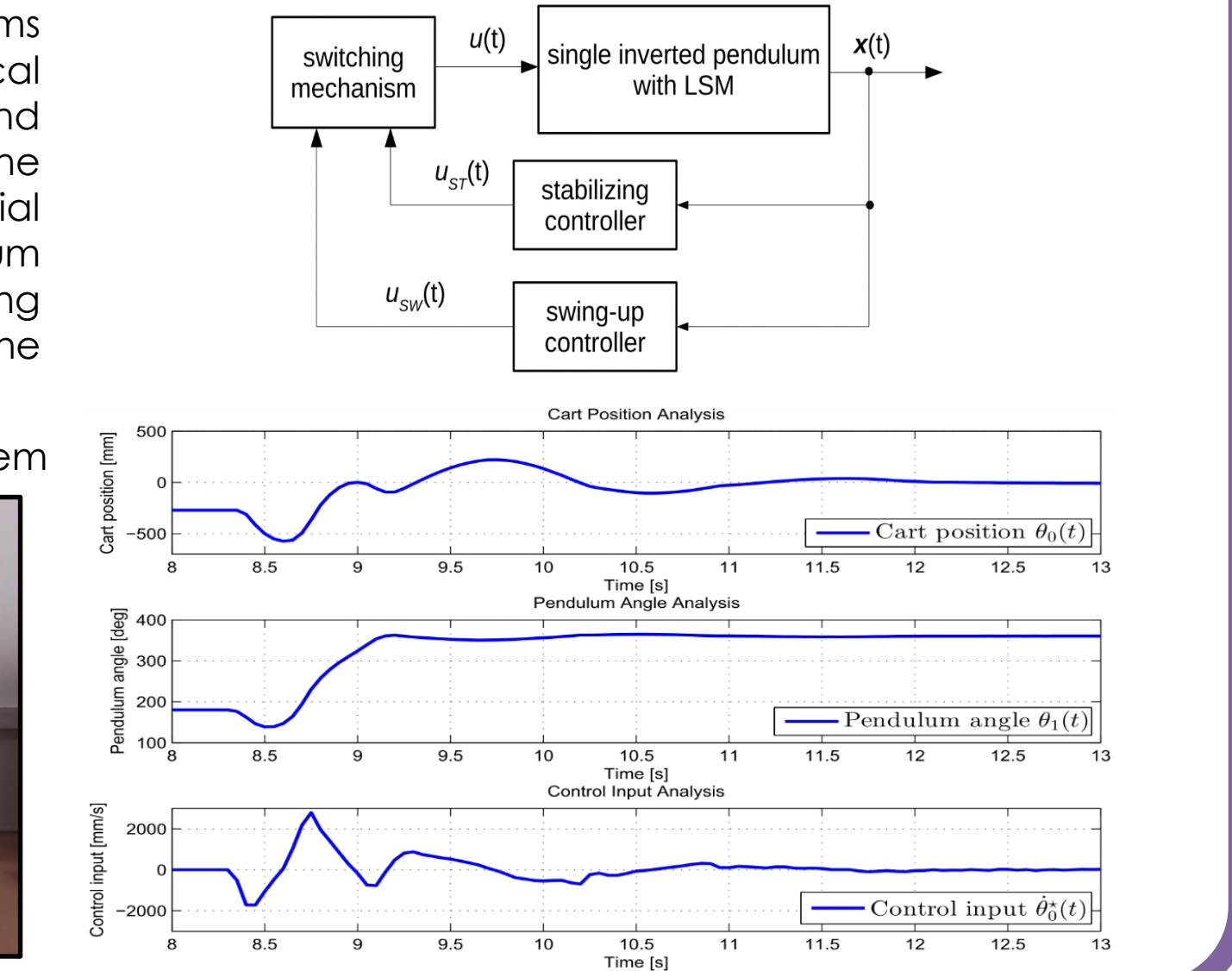


We have explored control problems of underactuated systems which require us to employ switching control structures. A typical example constitutes the hybrid control setup of a swing-up and balancing controller for selected benchmark systems, where the swing-up is performed via energy-based methods or partial feedback linearization. Laboratory models of inverted pendulum systems have enabled us to verify the swing-up and stabilizing control algorithms while also considering the properties of the actuating mechanism.

Laboratory model of a) classical b) rotary inverted pendulum system



Swing-up and LQR stabilization of a classical single inverted pendulum with a linear synchronous motor



Hybrid systems theory was developed to provide a convenient framework for modeling and control of systems characterized by an interaction between continuous (time-driven) and discrete (event-driven) dynamics. Hybrid models are useful if we have to consider an event-based description of the mechanical system dynamics, such as the configurations of legged walking robots. The Compass Gait is a principal example of an unpowered walking robot which performs gravity-induced passive motion on an inclined plane. This model can be gradually expanded to obtain a detailed walking robot model with natural dynamics.