Abstract — This paper deals with hybrid modelling and control of cyber-physical systems, which are part of a recently introduced phenomenon Industry 4.0. As a case study we look at a hydraulic system with hybrid dynamics represented by coupled tanks. The dynamical system contains two discrete modes, specifically with and without interaction. The nonlinear hybrid system was modelled using s-functions and then its linear description was created in the discrete PWA representation with the help of HYSDEL modelling framework. Both s-functions and HYSDEL are part of MATLAB/Simulink environment. After validation of the discrete PWA representation in comparison with the nonlinear hybrid system, LQR synthesis with reference trajectory tracking was designed for both discrete modes. Subsequently optimal control designed on the linear hybrid system in the discrete PWA representation was verified on the nonlinear hybrid system.

Keywords— cyber-physical system, hybrid system, discrete hybrid automata, coupled tanks, optimal control

CPS overview

What is a Cyber Physical System?
- tight integration of computational and physical resources

CPS challenges
- CPS composition
- Security and safety
- Hybrid modelling and control
- Architecture
- Sensor and mobile networks

Discrete Hybrid Automata

- switched affine systems - describe the continuous part of the hybrid system
- event generator - based on the fulfillment of boundary conditions provides a generation of a specific event
- finite state machines - represent the discrete part of the hybrid systems
- mode selector - based on the output of the event generator and finite state machine it defines the continuous part of the dynamical system

PWA Representation of the Hydraulic Coupled Tanks System

Optimal control design for hybrid systems requires linear representation of the nonlinear system in the PWA (SAS) form

Optimal Control of the Hydraulic Coupled Tanks System

Optimal control is based on minimizing cost function:
\[ J(k) = \sum_{i=1}^{N} \left( \Delta x_i(k)^T P \Delta x_i(k) + \Delta u_i(k)^T Q \Delta u_i(k) \right) \]
which solution is state-feedback gain \( k_c \)

\( N_c - \) feed-forward gain

\[ N_c = C_i (I - (F_1 - G_1 k_c))^i C_i^T \]

Control input signal:

\[ \Delta u_i(k) = -k_c \Delta x_i(k) + N_c \Delta x_i(k) \]

State-space partitioning

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