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# A Survey of Approaches for Modeling and Control of Effective Walking Robots

<sup>1</sup>Lukáš Koska (1<sup>st</sup> year),

Supervisor: <sup>2</sup>Anna JADLOVSKÁ

<sup>1,2</sup>Dept. of Cybernetics and Artificial Intelligence, FEI TU of Košice, Slovak Republic

<sup>1</sup>lukas.koska@tuke.sk, <sup>2</sup>anna.jadlovska@tuke.sk

**Abstract**—This paper includes a review of worldwide research in the area of robot locomotion, with the particular focus on modeling and control of underactuated biped robots. Since dynamic models of biped robots are hybrid in nature, the paper presents an overview of modeling the continuous and discrete dynamics of a hybrid system representing a walking robot. The paper briefly describes the design evolution in underactuated bipedal robots and presents several prominent prototypes, explains a number of related concepts and lists a variety of suitable control techniques. These topics will be elaborated on in the future dissertation thesis.

**Keywords**—hybrid systems, robot walking, underactuated systems

## I. INTRODUCTION

Reasons for building robots with human characteristics are various. One point of view is practical i.e., if robots are to work inside a house that was built for humans to assist humans. Examples include ASIMO or ATLAS [1] For similar reasons, NASA is developing Robonaut [2], an astronaut-size robot. As a second reason, various studies indicate that people are more likely to accept robots in their environment if they appear natural i.e. human-like [3]. Thirdly, from an engineering point of view, bipedal robots can be designed not just for the purpose of having human-like features but because human shapes have evolved to be the optimal solution for certain problem. For example humans have very articulate legs, so is much easier moving around in a rocky outdoor environment for humans than for cars [4].

One of the most interesting aspects of a humanoid robot is its locomotion technique: walking [5]. Walking can be defined as a locomotion gait of a biped, in which the feet are lifted alternately, while at least one foot is on the ground at all times. With bipedal robots we therefore speak of a swing and stance phase. If both feet temporarily leave the ground, the gait is called running [5].

The main way in which walking locomotion is implemented in robots is based on so-called *static walking*. If we construct a fully actuated robot, and ensure (by means of active control) that the center of mass is always located above the foot area, then if the robot is moving slowly enough, it is always stable. The control problem of walking is thus reduced to traditional joint tracking control [3]. These robots are fully actuated, because we have to control every degree of freedom but the disadvantage of this method is high energy consumption.

On the other hand, *dynamic walking* is much more energy efficient, as McGeer in [6] showed in his remarkable work on

passive walking robots, which are naturally capable of walking down an inclined plane. The center of mass of these robots does not always remain above the stance foot. Spong & Bullo [7] describe a control law that effectively rotates the apparent gravitational field, thus making the controlled robot move with the same gait on different slopes [5]. These robots can be modeled as underactuated, which means that we have fewer control inputs to the system than degrees of freedom.

This article deals with robotic locomotion with the focus of biped robots and provides a brief overview of design evolution. It provides the comparison between static and dynamic walking, the basics of the modeling a walking like a hybrid system and describes principal methods of controlling the robot locomotion.

## II. BRIEF OVERVIEW OF ROBOT LOCOMOTION

Several results show that underactuated bipedal robots with natural dynamics are more effective than fully-actuated robots. Mochon and McMahon in [8] proposed a mathematical model for human walking which incorporated biomechanics by assuming that an input is provided at the double-support only, with swing phase driven by only gravity. Research in underactuated bipedal robots got a boost with Tad McGeers Passive dynamic walkers in [6]. The passive walker reduces the motion to 2-dimensional, by designing the outer leg as a pair of crutches. The compass gait model based planar passive walker studied by Goswami et al. [9] consists of two stiff legs, connected by a passive joint at the hip. Tedrake et. al. [10] developed a simple 3D biped robot *Toddler*. McGeer in [11] added knees to his original model to overcome the foot scuffing problem without any actuation.



Fig. 1. Rabbit [12]

Several complex bipedal robot prototypes have been developed by research groups worldwide. *Rabbit* was created as a result of the collaboration of several French research

laboratories, from mechanics through automated control to robotics [12]. The emergence of Rabbit is linked to the study of the fully-actuated biped robot movement as opposed to passive movement Rabbit is shown in Fig 1.

*Ernie*, in Fig 2, was developed by Ryan Bockbrader from Ohio State University for research and education purposes. The main design features were inspired by the Rabbit robot. The legs used by Ernie are modular, which means that the length of legs, leg ends, and leg balancing can be changed by a minimal changes in design [12].

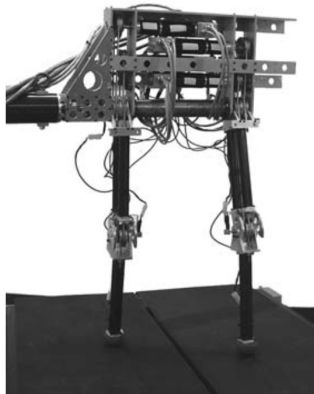


Fig. 2. Ernie robot [12]

The *Toddler* was designed by Russell Tedrake on MIT. The Toddler comes out of a simple robot compass gait design that has been expanded with curved feet. It has been designed to extend the simple compass gait robot with the actuators on legs, while the leg joints have remained passive [10]. It is shown in Fig. 3.

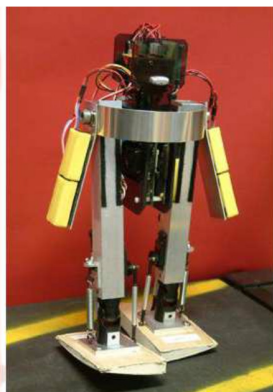


Fig. 3. Toddler robot [10]

*Atlas* is the most advanced humanoid robot well-known for its remarkable human-like gait in variety of conditions. Atlas' control system coordinates motions of the arms, torso and legs to achieve whole-body mobile manipulation. Moreover Atlas keeps its balance when jostled or pushed and can get up if it tips over [1]. Atlas is shown on the Fig. 4.

Positioning the actuators into the torso of the biped reduces the weight, that is placed off of the robot mass center. The result is lighter legs that allow the use of weaker actuators [12]. This approach requires more complex gearboxes. Actuators can be position controlled servo-motors and allow to simulate fully passive walking along the inclined plane [3].

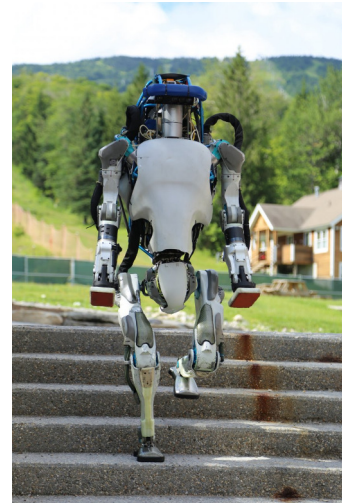


Fig. 4. Atlas robot [1]

All mentioned walking robots require an accurate mathematical model, which is represented as a hybrid dynamical model.

### III. APPLICATION OF HYBRID SYSTEMS THEORY IN MODELING OF WALKING ROBOTS

Bipedal walking is a repeated sequence of steps made by an alternating pair of legs. At each step, there is a phase where the swing leg comes into contact with the ground. As a result, dynamic models for bipeds are hybrid in nature. They contain both continuous and discrete elements, with switching events that are governed by a combination of one-sided constraints and impulse-like forces that occur at foot touchdown [13].

#### A. Hybrid dynamic

The hybrid system can be described by hybrid state vector, which is analogical to the continuous-state vector. Hybrid state vector  $\zeta(t)$  consists from state vector  $\mathbf{x}(t) \in \mathbb{R}^n$ , which is the same as in the continuous systems and the discrete variable  $x_d(t) \in \mathbb{Z}$ . The hybrid vector itself can be defined as:

$$\zeta(t) = \begin{bmatrix} \mathbf{x}(t) \\ x_d(t) \end{bmatrix} \quad (1)$$

Inputs to the hybrid system are analogical to hybrid states and can be divided to continuous external inputs  $\mathbf{u}(t) \in \mathbb{R}^m$  and discrete external inputs  $u_d(t) \in \mathbb{Z}$ . Continuous inputs affects directly continuous dynamic and discrete inputs affects discrete dynamic [14]. Outputs from hybrid system are also continuous  $\mathbf{y}(t)$  and discrete  $y_d(t)$  and they are determined by output function  $\mathbf{h}(\mathbf{x}, \mathbf{u}, x_d, u_d, t)$ .

#### B. Continuous dynamics

As is clear from the definition of hybrid systems, each discrete state is described by its own continuous dynamics. In the case of mechanical systems including underactuated locomotion systems, this results in the use of the Lagrange equations for each state [15].

Difference of the kinetic and potential energy or the Lagrangian [10], can be expressed as the following function of generalized coordinates  $q(t)$  and their velocities  $\dot{q}(t)$ :

$$L(q(t), \dot{q}(t)) = E_k(q(t), \dot{q}(t)) - E_p(q(t)) \quad (2)$$

where  $E_k$  is the overall kinetic energy and  $E_p$  the overall potential energy of the system [16].

Given that  $Q^*(t)$  is the vector of external forces acting on the system, Lagrange equations of the second kind are specified by the relationship:

$$\frac{d}{dt} \left( \frac{\partial L(t)}{\partial \dot{\mathbf{q}}(t)} \right) - \frac{\partial L(t)}{\partial \mathbf{q}(t)} = \mathbf{Q}^*(t) \quad (3)$$

and yield the set of motion equations describing the dynamics of a given mechanical system [15]. It is customary in mechatronics and robotics to rewrite the equations of motion into the so-called standard minimal form:

$$\mathbf{M}(\mathbf{q}(t))\ddot{\mathbf{q}}(t) + \mathbf{N}(\dot{\mathbf{q}}(t), \mathbf{q}(t))\dot{\mathbf{q}}(t) + \mathbf{P}(\mathbf{q}(t)) = \mathbf{V}(t)\mathbf{u}(t) \quad (4)$$

where  $\mathbf{M}(t)$  is the inertia matrix,  $\mathbf{N}(t)$  is the Coriolis force matrix,  $\mathbf{P}(t)$  is the matrix of potentials, and matrix  $\mathbf{V}(t)$  maps inputs of  $\mathbf{u}(t)$  to generalized coordinates. If the rank of matrix  $\mathbf{V}$  is equal to the dimension of the vector of generalized coordinates, the system is fully actuated, if  $\text{rank}(\mathbf{V}) < \text{dim}(\mathbf{q})$  the system is underactuated [17]. The continuous state vector is then defined as  $\mathbf{x}(t) = [\mathbf{q}(t) \quad \dot{\mathbf{q}}(t)]$ .

### C. Discrete dynamics

The discrete dynamics making up the second part of overall dynamics of the hybrid system is defined by the existence of events specified by the extended hybrid state vector  $\zeta(t) = (\mathbf{x}(t), \mathbf{u}(t), x_d(t), u_d(t))$ . At the time that the extended state vector enters the  $S_i$  switching space, the event is generated.

Using switching maps  $\Phi_i(\mathbf{x}, \mathbf{u}, x_d, u_d, t^-)$ , there is discontinuity at the moment when the event occurs and a jump change occurs [10]. The hybrid vector at the time of the generated event is determined by the appropriate switching map:

$$\zeta^+ = \Delta_i(\mathbf{x}, \mathbf{u}, x_d, u_d, t^-) \quad (5)$$

The generated event at time  $t$  may also mean switching between continuous dynamics and changing the size of continuous state vector  $\mathbf{x}$ .

Models of this form are very useful for simulating bipedal robot locomotion. Complementarity Lagrangian models can be also used for trajectory optimization, without a priori enumeration of the type and order of the contact events [13].

### D. Benchmark systems for the analysis of robot locomotion

Human locomotion stands out among other kinds of double-sided movement mainly because during walking the moving human body is out of the static equilibrium [18]. This is the reason why it is useful to explore natural two-legged gait first in a simplified perspective, and then gradually expand the simpler models to get closer to human walking [19]. The simplest underactuated walking models are derived from underactuated benchmark systems such as a double pendulum or the Acrobot whose dynamics is analogous to the continuous dynamics of the biped walker [10].

Several hybrid benchmark systems for robot walking include the following [20]:

Compass Gait is the basic passive walking model with two degrees of freedom. It is composed of a pair of rigid

Fig. 5. Compass Gait and Kneed Walker

bars, representing legs without knee and foot, connected by a frictionless hinge at the hip, see Fig. 5. The robot is underactuated and able to walk down a plane inclined under a given angle. [21]. This model was the basis for Toddler robot by R. Tedrake [10].

Walking with knees is a natural extension of the compass gait model. Addition of knees increases the number of degrees of freedom in the system, and at the same time, eliminates the foot trapping on the ground [20].

The kneed walker model has three degrees of freedom, which can be described by the following vector of generalized coordinates consists of the rotation angle of the stance leg  $\theta_1(t)$ , the angle of the swing leg  $\theta_2(t)$  and the angle of the tibia part  $\theta_3(t)$  [22]. The continuous dynamics of the kneed walker differs from that of the compass gait in that it includes two dynamical models, namely the dynamics with unlocked knees and locked knees [4].

3D Compass Gait model is an extension of the simple compass gait model. Extensions consist of adding the curved feet and adding the frontal plane which replace the leg switching of the simple compass gait. Firstly, the addition of the large curved feet dramatically increases the basin of attraction of the stable walking solution. Secondly, the point masses in the compass gait model can be replaced with the more realistic mass and moments of inertia for each link. The mechanism for achieving foot clearance on this walker is a gentle rocking motion in the frontal plane. To model these dynamics, it is assumed that the robot is always in contact with the ground at exactly one point and that the foot rolls without slipping [10].

These models are usually simulated with effective simulation tools, such as *MATLAB/Simulink* and *Stateflow* toolbox for modeling the discrete dynamics.

## IV. CONTROL OF ROBOTIC LOCOMOTION

One of the most common control methods for bipedal locomotion is the Zero Moment Point (ZMP) control strategy. The ZMP is the point on the ground at which the reaction forces acting between the ground and the foot produce no horizontal moment. Under this condition on the ZMP, the stance foot remains at on the ground and immobile [13]. Early implementations of ZMP-based controllers used offline trajectory optimization to generate center of mass trajectories [23]. Other methods have achieved improved control by formulating the control problem as a quadratic program as shown by Feynman in [16].

Because the ZMP algorithm is suitable only on the fully-actuated robots, control of underactuated robot locomotion requires other approach of the control such as partial feedback linearization and hybrid zero dynamics [4]. Since the



underactuated systems are not feedback linearizable, part of the dynamics corresponding to the actuated degrees of freedom is linearized with a nonlinear feedback. This method was used by Tedrake in [24].

The method of hybrid zero dynamics consists of defining a set of outputs, equal in number to the inputs, and then designing a feedback controller that asymptotically drives the outputs to zero [25]. The task that the robot is to achieve is encoded into the set of outputs in a such a way that the nulling of the outputs is (asymptotically) equivalent to achieving the task, whether the task be asymptotic convergence to an equilibrium point, a surface, or a time trajectory [12]. Sreenath demonstrated that the method of hybrid zero dynamics gave good results on robot Mabel in [26]. Other control approaches used for stabilizing bipedal walking are: passivity based control [10], adaptive predictive control [21], transverse linearization [27], dynamical optimization, event-based control, time-based control, AI methods or intelligent learning control [4].

Direct methods for trajectory optimization are widely used for planning locally optimal trajectories of robotic systems. The technique treats the discontinuous dynamics of contact as discrete modes and restrict the search for a complete path to a specified sequence through these modes [10].

State dependent Riccati equation which is similar to LQ control law, but the feedback gain is online recalculated according to state vector [28].

Boundary value problem leads to the offline generation of optimal robot and of course joint trajectories [17]. The role of research is to define correct boundary functions and sensitivity functions to achieve periodic trajectories.

## V. CONCLUSION

This paper provides a basic notion of passive robotic walking. Underactuated passive gait is of a great importance in the research of anthropomorphic robotic walking and a detailed and accurate mathematical model of a biped robot with given features is a necessary precondition for any kind of analysis and control algorithm design. In my thesis Design of a Methodology for Modeling and Simulation of Effective Walking Robotic Systems using Modern Simulation Tools and the Industry 4.0 Concept. I will focus on modeling of complex gait models and the design and implementation of suitable control algorithms. Because of complexity of the design of robotic gait, the considered algorithms are first applied to simpler benchmark models like inverted pendulum system, Acrobot or Pendubot.

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