Underactuated Mechanical Systems: Recent Findings and Future Challenges

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***Abstract*— This paper describes the current state of our research in the area of modeling and optimal control of underactuated mechanical systems. Emphasis is placed on the results obtained during the past year. Plans and goals for the following period of time are briefly outlined.**

***Keywords*—underactuated mechanical systems, Lagrangian mechanics, optimal control techniques, hybrid systems theory**

I. Introduction

Underactuated systems represent a significant group of mechanical systems which range from simple planar robots and inverted pendulum systems to advanced higher-order systems such as mobile robots and manipulators, aircraft and watercraft vehicles. Such systems are inherently nonlinear and have fewer control inputs than degrees of freedom. A number of strong structural properties which facilitate control design for fully actuated systems (e.g. feedback linearizability, passivity, linear parametrizability) are usually lost in underactuated systems, while at the same time undesirable properties (higher relative degree, nonminimum phase behavior) emerge [1].

As it was formulated in the February 2013 proposal, the general objective of my PhD thesis, titled *Modeling and Optimal Control of Nonlinear Underactuated Dynamical Systems*, is to identify and then tackle the open research problems occurring at the mutual overlaps between three principal areas: Lagrangian dynamics of underactuated mechanical systems, optimal control techniques, and hybrid systems theory. The presented paper details which aspects of this objective have so far been accomplished and to what extent, and describes the algorithms and supporting tools which were developed to achieve selected tasks.

II. Inverted Pendulum Systems – A Generalized Approach to Modeling

Principles of modeling and control of inverted pendulum systems (IPSs) are generally considered as the basic starting point for the research of advanced underactuated systems [1]. In the initial phases of our research, we decided to focus on the development of algorithms for automated Lagrangian model generation. Based on the mutual analogy between mathematical models of IPSs with a varying number of pendulum links, we introduced the concept of a *generalized n-link inverted pendulum system* with $n+1$ degrees of freedom and a single actuator, which allows to treat an arbitrary IPS as a particular instance of the system of n pendula attached to a given stabilizing base, such as a cart or a rotary arm. General procedures which determine the Lagrange equations of motion for a user-specified instance of a generalized classical and rotary IPS were developed and implemented via MATLAB's *Symbolic Math Toolbox*. In the journal paper [2], physical formulae which make up the core of both procedures were derived and presented together with generated motion equations of example IPSs and verification of their validity. Case studies devoted to modeling and control of the classical double and the rotary single IPS were published as [3][4]. All simulation experiments were conducted using pre-prepared blocks from a *Simulink* block library which was developed as a software framework for the analysis and control of IPSs [5].

III. Recent Advances in Modeling and Control of Underactuated Systems

The past year's research proceeded in several directions. First of all, a general survey paper devoted to the main achieved results in modeling and control of underactuated systems was prepared [6]. After a summary of mathematical/ physical preliminaries, we presented an overview of principal categories of underactuated systems, evaluated the ability of optimal control techniques to suit the properties of underactuated systems, and examined the potential of hybrid systems theory (i.e. integration of continuous and discrete dynamics in a dynamical system) with regard to modeling and control of underactuated systems [7]. The paper was awarded the 1st Dean's prize for the best oral presentation in the IT section of the conference and was recommended for journal publication, which is currently under preparation.

As a direct follow-up to our previous work on automated modeling, we expanded the existing procedure for automatic model generation of classical IPSs with further practically motivated generalizations. The expanded general procedure now covers all feasible combinations of initial assumptions for the pendulum reference position and direction of rotation, and considers various shapes of weight load attached to the last pendulum link. The design and program implementation of the procedure is detailed in [8]. Existing library blocks were updated as well: the necessity to create a separate block for each set of equations was eliminated by implementing dynamic adjustments of block structure, so that it would always correspond to a specific set of motion equations.

The rest of our efforts focused on control algorithm design. The rotary single IPS case study paper [4] was rated as one of the top 10% papers presented at the conference and was selected for publication in a journal. The extended version [9] presents a complex problem of swinging up and subsequent stabilization of IPS in the unstable position, which involves a hybrid setup of a swing-up and balancing controller in a switching control structure. The stabilization problem was solved via optimal control algorithms based on quadratic functional minimization using continuous-time and discrete-time linearized state-space models; additional control structures were introduced which ensure that the rotary arm position reaches the reference value, and the permanent steady-state error is eliminated. Swing-up of the pendulum was performed via energy-based methods.

In [10], we moved on to examine the nonlinear control design technique based on the *state-dependent Riccati equation* (SDRE) which creates a separate linear quadratic optimal control problem at each time step using the original nonlinear state-space description of the system. The algorithm of SDRE control design was generalized for an *n-link* IPS, and verified on a rotary single inverted pendulum model. The results were compared to standard LQR control algorithm and it was shown that the SDRE-based controller, which preserves the original nonlinear system dynamics, gradually outperforms the conventional discrete-time LQR algorithm if the distance increases between the state-space vector and the equilibrium.

The findings presented in this paper and in referenced works make up a total of three chapters of a recently published monograph [11] supported by a KEGA project. The chapters respectively deal with modeling of underactuated mechanical systems using Lagrangian mechanics [2][3][4][5][8], optimal/predictive control techniques and their application in control of underactuated systems [3][4][6][9][10] and hybrid systems theory, notably switching control structures [6][9]. Each chapter contains theoretical derivations, presentation of developed software tools, and a number of simulation results.

IV. Conclusion and Plans for Future Research

This paper summarizes the results we have achieved in the area of underactuated mechanical systems. The principal aim of our current research is to identify and overcome the critical aspects of control algorithm design caused by certain disadvantageous physical properties of underactuated systems. With linear quadratic optimal control experimentally confirmed as a reliable control technique for a class of underactuated systems; we now intend to thoroughly analyze the required adjustments to model predictive control algorithms. Promising application areas of hybrid systems theory include hybrid models for underactuated systems with logical parts, hybrid optimal/predictive control algorithms and switching control structures. Meanwhile, algorithms for automated model generation are being developed for typical representatives of underactuated robotic systems, such as the *Acrobot/Pendubot* and *reaction wheel pendulum*. The ultimate goal is to prepare a readily available collection of mathematical and simulation models of underactuated mechanical systems which should serve as a testbed model basis in simulation experiments for exploring their properties and testing linear and nonlinear control strategies.

In order to contribute to the modeling and control education at the DCAI-FEEI TU, nonlinear underactuated systems are being integrated into the research and teaching activities of the *Center of Modern Control Techniques and Industrial Informatics* at the DCAI. In addition to the already developed library of simulation models, a modular inverted pendulum model is currently being constructed and several other options for a laboratory model of an underactuated system are being considered. Collaboration possibilities with other centers at the DCAI are being explored. These include the development of modeling and control techniques for biped robots, which were so far studied by the *Center of Intelligent Technologies*.

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