# **CLASSICAL DOUBLE INVERTED PENDULUM - A COMPLEX OVERVIEW OF A SYSTEM**

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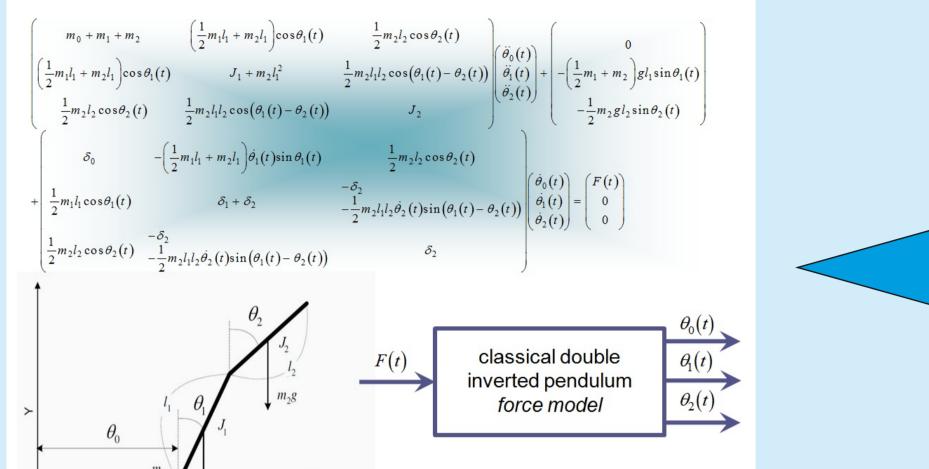
- mechanical system composed of a pair of rigid, homogenous pendulum rods interconnected in a joint; one of these is attached to a stable mechanism (cart) which allows for movement alongside a single axis - a typical example of an underactuated system - the only input (force acting upon the cart) is used to control the three degrees of freedom: cart position,

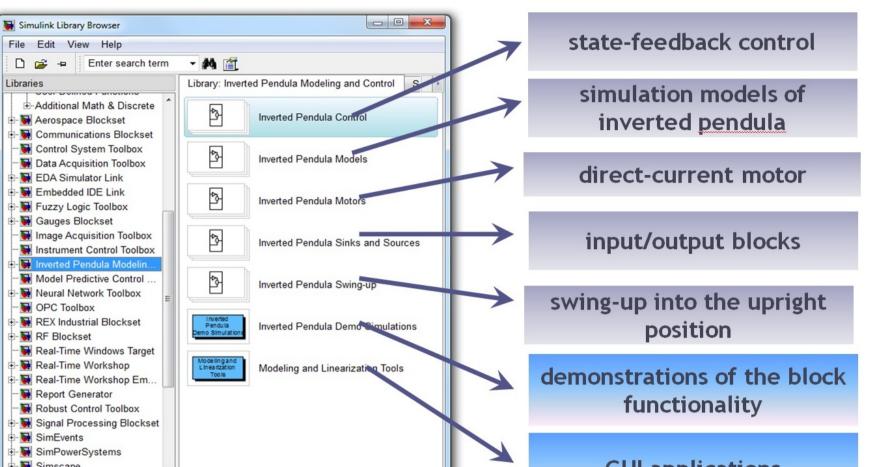
lower pendulum angle and upper pendulum angle

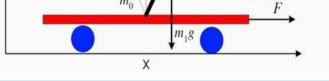


- Inverted Pendula Modeling and Control (IPMaC)
  structured, thematic Simulink block library (extended & improved 2009 version)
  comprehensive software framework for the problems of analysis & control of inverted pendula systems

- supports classical & rotary inverted pendula systems
- strong emphasis on the generalized approach to modeling
   contains custom function blocks, GUI applications and a set of links to demo simulation schemes







Showing: Inverted Pendula Modeling and Control	GUI applications	

Inverted Pendula Model Equation Derivator - generates the mathematical model (equations of motion) for a user-chosen type of inverted pendula system (classical double inverted pendulum system selected)

- implements a general procedure based on the the Lagrange equations of the second kind, which derives the motion equations for a generalized (n-link) classical and a generalized rotary system of inverted pendula

Inverted Pendula Model Equation Derivator Inverted pendula system type Derive model equations		initiates the process of mathematical model derivation		
Number of pendula: two (double inverted pendulum)  Type of system Classical Totary	Classical Double Inverted Pendulum Motion Equations Parameters: F - input force m0 - cart mass, m1 - lower pendulum mass, m2 - upper pendulum mass I1 - lower pendulum length, I2 - upper pendulum length delta0 - cart friction, delta1 - lower pendulum damping, delta2 - upper pen JT1 - lower pendulum moment of inertia, JT2 - upper pendulum moment of fi0 - arm angle, fi1 - lower pendulum angle, fi2 - upper pendulum angle, dfi pendulum angular velocity Model equations Cart equation:	finertia		
	<pre>delta0*dfi0 + d2fi0*(m0 + m1 + m2) - (m1*(dfi1^2* (m2*(2*11*sin(fi1)*dfi1^2 + 12*sin(fi2)*dfi2^2 - 2* Lower pendulum equation: (11*12*m2*sin(fi1 - fi2)*dfi2^2)/2 + dfi1*(delta1</pre>	<pre>d2fi1*l1*cos(fi1) - d2fi2*l2*cos(fi2)))/2 = F - delta2) + d2fi1*(JT1 + (l1^2*m1)/4 + n2*cos(fi1) - (g*l1*m1*sin(fi1))/2 - = 0</pre>		
x	- (l1*l2*m2*sin(fi1 - fi2)*dfi1^2)/2 + delta (d2fi0*l2*m2*cos(fi2))/2 - (g*l2*m2*sin(fi2))/2			
		generated motion equations		
Ор	en-Loop Dynamic	cal Analysis		

fi0 cart positi

Simulation time [s]

## Inverted Pendula Model Linearizator & Discretizer

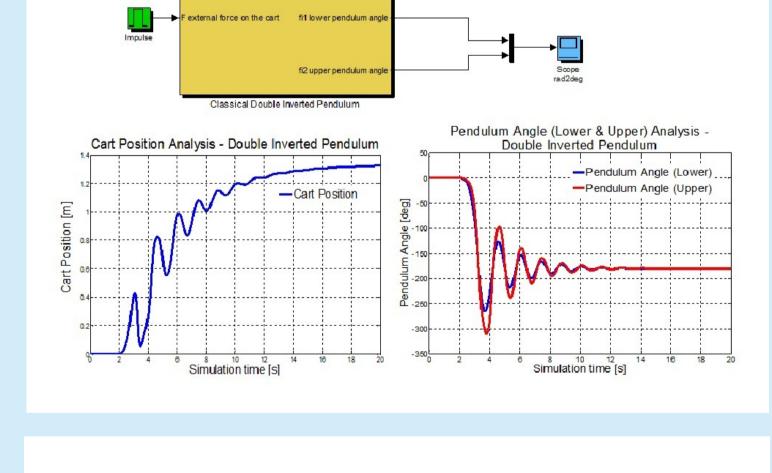
- generates the state-space matrices of the continuous-time linear approximation of the system in a selected equilibrium & also returns the matrices of the discrete-time state-space model if the sampling period constant is known

- performs the standard expansion of the symbolic nonlinear state-space description of the system into the Taylor series with higher-order terms neglected

Inverted endula Model Linearizator & Discretizer Inverted Pendula Mc del Linearizator & Discretizer Inverted pendula system type Number of pendula:	Model parameters Cart mass (m0): 0.5 Lower pendulum mass (m1): 0.27 Upper pendulum mass (m2): 0.27	5 Lower pendulum length	Friction coefficient (delta	a0): 0.3 wer (delta1): 0.1	system parameters
two (double inverted pendulum)  Type of system Classical Totary	Motor parameters	Motor torque constant (K Gearbox ratio (Kg): Lower pendulum positio	3.7 Motor pinion radius (r):	2.6 0.00635	motor parameters
	Continuous-time state-space matrices State matrix A: 0 0 0 0 0 0 0 0	Input 1 0 0 0 1 0 0 0 1 0 0 1 0 1.0370 -0.5926 5 -11.3939 7.7576 -7.6	matrix B: Output matrix C: 0 1 0 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1		matrices of the linearized system
Bo F	Discretize system           Discrete-time state-space matrices           State matrix F:           1 -4.1099e-04 3.7399e-05 0.0093           0 1.0034 -0.0013 0.0016           0 -0.0038 1.0032 -4.4262e-04           0 -0.0786 0.0063 0.8729           0 0.6544 -0.2383 0.3101           0 -0.7149 0.6137 -0.0751           4	4.4770e-05         -2.5723e-05         1.403           5         0.0095         3.4974e-04         -3.49           8.2989e-04         0.0094         9.53           0.0083         -0.0048         -0.0048           0.9034         0.0664         -	Imatrix G:         Output matrix C:           58e-04         1         0           15e-04         0         1         0           16e-05         0         1         0           0.0274         0         1         0           0.0668         0         1         0           0.0162         0         0         1	0 0 0 0 0 0 0 0 0 0 0 v trix D:	matrices of the discretized system

## State-Feedback Control Examples

- LQR control (optimal state-feedback control) of a voltage model of a classical double inverted pendulum system; comparison of the performance of a standard LQR controller and a controller with a summator included in the control structure (permanent disturbance elimination)



### classical double inverted pendulum system

- simulation scheme and time behavior of the cart position and pendula angles

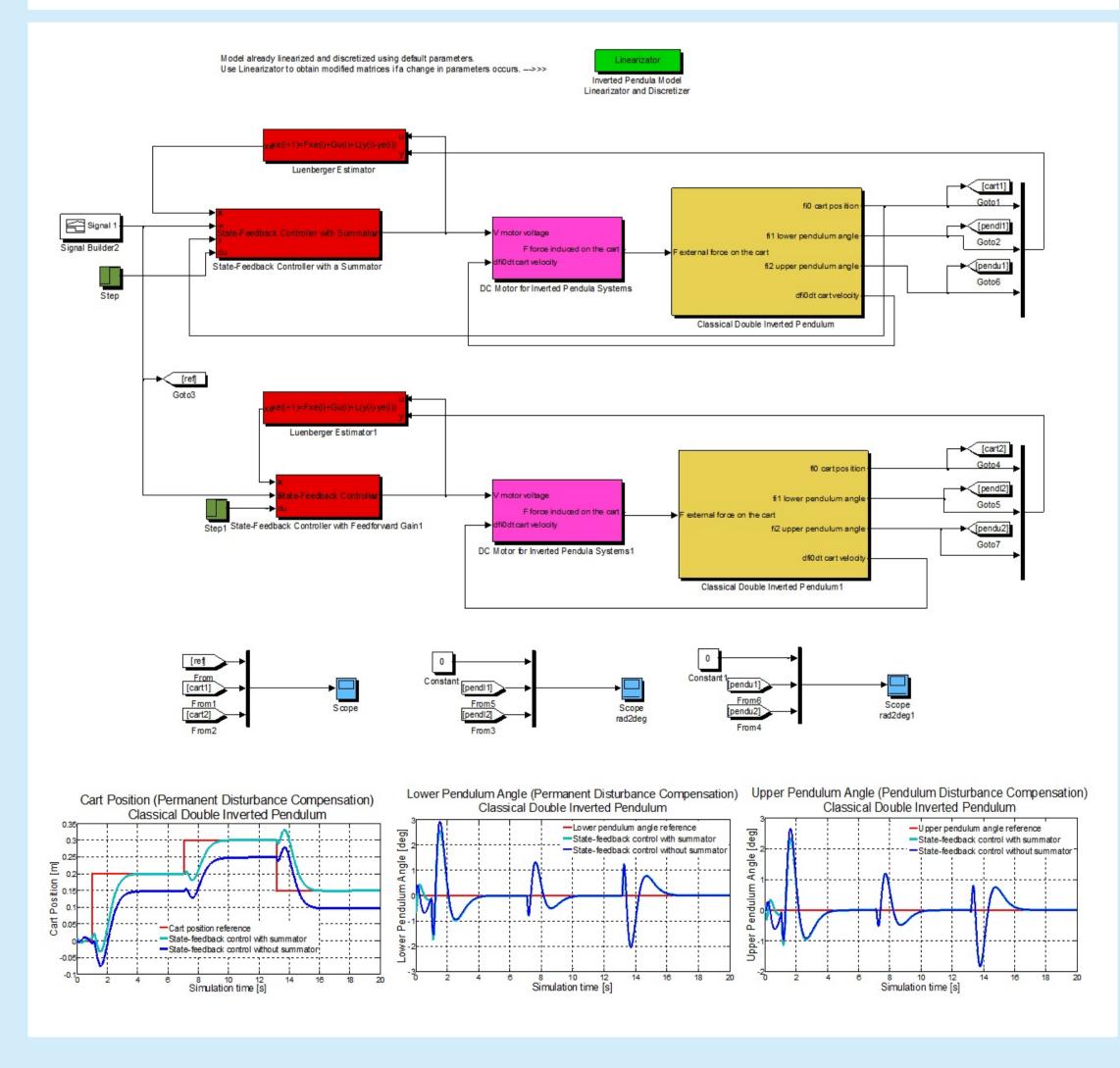
- the scheme is located in the *Analysis 1* / Nonlinear Analysis section of the Inverted Pendula Demo Simulations

- open-loop response to an impulse signal for a **voltage model** of a classical double inverted pendulum system (i.e. a model of a DC motor is attached to the inverted pendulum system to provide the force which actuates the cart and both pendula)

- simulation scheme and time behavior of the cart position and pendula angles

- the scheme is located in the *Analysis 3* / Nonlinear Analysis with DC Motor section of the Inverted Pendula Demo Simulations

- simulation scheme and time behavior of the cart position and pendula angles - the scheme is located in the *Control Algorithm 2 / Summator Control* section of the *Inverted Pendula* Demo Simulations.



Scope2 fi1 lower pendulum and fi2 upper pendulum and external force on the cart dfi0dt cart velocit DC Motor for Inverted Pendula System i1dt lower pendulum angular velocity 2dt upper pendulum angular velog rad2deo Classical Double Inverted Pendulun Cart Position Analysis Pendulum Angle (Lower & Upper) Analysis Classical Double Inverted Pendulum with DC Motor Classical Double Inverted Pendulum with DC Motor -Pendulum Angle (Lower) Cart position

Simulation time [s]

