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Utilization of Petri nets for the analysis of production systems

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Abstract

This article deals with the use of Petri nets for the analysis of production systems. The aim of the article is present analysis options of production systems at the stage of design using Petri nets. The article deals with the calculation of the reachability of required state of the production system. There exists the firing sequence for every reachable state to transform the system from the initial to the required state. Firing sequence allows the analysis of duration of transformation in production system to the required state, the performance of the production system, as well as production costs.

Keywords: Petri net, production system, incidence matrix, firing sequence

Nomenclature

M_0	initial marking of Petri net
M_x	desired state of Petri net
C	incidence matrix
C^{-1}	inverse matrix C
t	Petri net transformation time
p	performance of production system
v	percentage performance
k	Petri net transformation cost
F	cost vector
P	set of place
T	set of transition
pre	direct incidence matrix
post	back incidence matrix
Tempo	time vector

Greek symbols

σ	firing sequence

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1. Introduction

The production systems at present are different from those older and especially the complexity of the technology used. The current trend is to have production that is fully automated which increases reliability, quality, productivity and efficiency. Major reasons for the automation of production systems include reduced time of production and reduced cost of production. During the design of production systems are difficult to analyze the proposed project. Analysis of the project design phase is difficult, but suggestions for improvement in the project simply completed. In analyzing the facts state bring better results, but completion of the project is usually difficult. The main terms that can be analyzed include the production cycle time and performance of production system. If performance is inadequate already at the design stage, it is probable that the project will not meet required standards. Adverse parameter is needed to prevent early analysis of production systems.

The first part is devoted to the description of methodology by which it is possible to analyze the production system, which is modeled by Petri nets (PN).

The second section briefly describes the actual model production line which will be validated methodology presented in the first part.

2. Analysis of the production system using Petri net

The analysis model of the production system in the design phase, or have a real system, it is possible to identify some of the required parameters. PN analysis which modeled the production system, it is possible to analyze the reachability of state PN, time for transformation to the desired state, the performance of the production system or the production costs.

2.1. Reachability state in Petri net

Reachability state in PS can be detected from the existence of a firing sequence. If a firing sequence exists, which transforms the PS from the initial to the desired state, then the desired state is achieved. The firing sequence can only take nonnegative values of whole numbers because the PS transition is carried out or not. Calculation of firing sequences based on the description of state equation, which has the shape (1).

$$\boldsymbol{M_x} = \boldsymbol{M_0} + \boldsymbol{C} * \boldsymbol{\sigma} \tag{1}$$

where $\mathbf{M_x}$ – is desired state,
$\mathbf{M_0}$ – is initial marking,
$\mathbf{C}$ – is incidence matrix,
$\boldsymbol{\sigma}$ – is firing sequence.

Equation in this form refers to the calculation of the desired state with a known firing sequence. However, the firing sequence for production systems is often not known. Known is desired state in which it is necessary to transform the PN, which corresponds to the real state of the production process.

In order to calculate a firing sequence, a state equation (1) must be modified to have the shape (2).

$$\boldsymbol{\sigma} = \boldsymbol{C}^{-1} * (\boldsymbol{M_x} + \boldsymbol{M_0}) \tag{2}$$

Existence of a firing sequence refers to the reachability of PN state which corresponds to the real state of the system.

This makes calculation of the reachability of any desired state possible. Tools for modeling PN imply a listing of all reachable states, but for large networks there is a high number of reachable states. It is sometimes difficult to find the desired state out of the high number of states. Therefore this method is replaced by searching the desired state through calculation.

2.2. Transformation time in Petri net

The production process time for a desired reachable state can be determined from the matrix **Tempo** in timed PN and a firing sequence for a desired state of PN.

Time required to transform PN to a desired state can be calculated using the equation (3).

$$t = \boldsymbol{Tempo} * \boldsymbol{\sigma} \quad (3)$$

When considering cyclically the transition of the network from the initial state to the end state, it is possible to calculate the ideal performance per hour of the production system according to the equation (4).

$$3600 / t = p \quad (4)$$

where p – is the number of transitions made per hour.

Real production process has certain requirements to operate. The machine operator is the activity during the production process does not work. For example: debugging, complete lack of material in storage, maintenance, production process, waiting for the essential components and the like. Perfect hourly output of each manufacturing process of these reasons, actually decreases. Generally, a reduction of ideal hourly performance is 20% to 30% in some production processes and more.

It is therefore necessary to calculate the real output per hour, which represents 70% to 80% of ideal hourly performance of the production process.

To calculate real hourly performance is required equation (4) modified as follows

$$3600 / t * (v / 100) = p \quad (5)$$

where v – is a percentage of real performance of production process.

The methodology for calculating the time transformation of PN is valid for production a system in which is processed single product.

2.3. Transformation cost in Petri net

With a firing sequence it is possible to determine the cost of the transformation of PN from the initial state to the desired state, the cost of one transition network.

The cost to transform the network is calculated using the equation (6).

$$k = \boldsymbol{F} * \boldsymbol{\sigma} \quad (6)$$

where k – is Petri net transformation cost
F – is cost vector

Careful analysis of those characteristics can determine reachability of various required states. The desired state is then analyzed and the results of the analysis are used to construct or modify production systems to meet the required properties.

Analysis of production time and of production cost help optimize the production process. Optimization can be done by preserving the whole structure and exchanging the components, which will bring better results. Alternatively, the production system can be extended with additional features or some parts can be omitted.

3. Description of the production system

This section briefly describes the production line shown in Fig. 1, which is used to experimentally verify the methodology described above. The second subsection is devoted to the description of Petri nets which model the mentioned production line.

3.1. Description of a flexible production system

The production system is used for teaching and a research purpose is the model of fully automated production line which was called a flexible production system (FPS). It is located at the Department of Cybernetics and Artificial Intelligence (DCAI), Technical University (TU) of Košice. The model is placed in the laboratory V147 which is situated in Vysokoškolska street number 4. The FPS is shown in Fig.1. The production line consists of six posts. Five posts make up the production cycle and the sixth post is separate and serves only to the replenishment of storages. The production cycle begins and ends on the third post. Postmarked is preserved from the design phase and therefore the production cycle does not begin on post one. On the post three there are templates prepared for production and also manufactured templates are stored. Prepared template on the post four moves to the second conveyor and then to post five. On the fifth post, a template is emptied and ready to be folded. On the first post, a required picture is folded. When a picture is folded, the template moves on to post two. On the second post, a camera check is performed and then template moves to the first conveyor. Template travels on the conveyor to the post three where it is stored in the warehouse and a new template is prepared. The sixth post is separate from the production cycle and is used for sorting colored cubes. A detailed description of FPS can be found in [1-3].

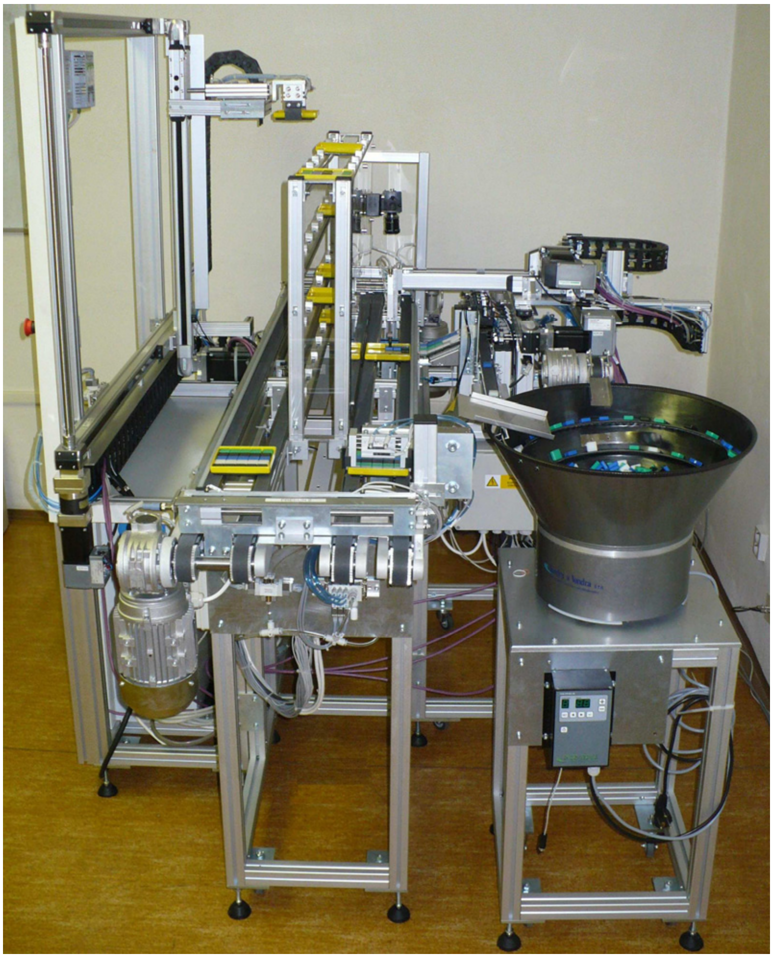

Fig. 1. Flexible production system

3.2. Description of a flexible production system using Petri net

FPS model is modeled by T - timed PN which is shown in Fig.2. This model describes basic activities of FPS. Detailed activities are not included in this model due to simplification of analysis. Simplified model does not affect the analysis options and a presented approach can also be used for more complex models. This FPS model will show basic calculation that must be carried out to analyze the PS which model the real production system.

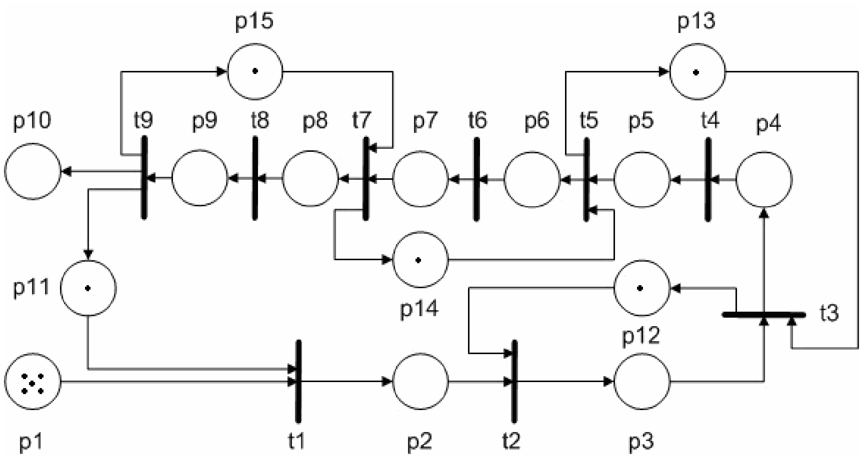

Fig. 2. Petri net modeling the flexible production system

Places and transitions in PN represent these states and activities of FPS:

p1 – the input storage of pallets
t1 – preparing for the production of the post 3, choice pallet from storage
p2 – pallet is on input storage of post 3 and waiting to prepare the post 4,
t2 – transferring pallet on the post 4,
p3 – a pallet of post 4 is ready to transfer to second conveyor and waiting to prepare the post 5,
t3 – transfer pallet from first conveyor to second conveyor and then on post 5, prepare the post 4,
p4 – a pallet of post 5 is ready to be emptied,
t4 – emptying pallet on the post 5,
p5 – empty pallet on the post 5 is ready to be transferred and is waiting to prepare a post,
t5 – transfer pallet of post 1, ready to post 5,
p6 – a pallet is on the post 1 and waiting for a folding shape,
t6 – folding pictures in a pallet of post 1,
p7 – a full pallet of post 1 is ready to be moved and waiting to prepare the post 2,
t7 – transfer a pallet to post 2, prepare a post 1,
p8 – a pallet of post 2 is waiting for the end of control by camera,
t8 – control by camera for a pallet of post 2,
p9 – a pallet after control by camera on the post 2 is ready to be moved to the output storage on post 3,
t9 – transfer pallet to the output storage on post 3, prepare the post 2,
p10 – a pallet is in the output storage on post 3,
p11 – production line prepared for the next pallet,
p12 – post 4 is ready
p13 – post 5 is ready,
p14 – a post is ready,
p15 – post 2 is ready.

PN that models the production system must satisfy certain properties of PN such as reachability, liveliness, conflictless and must not contain its own cycles. As early as in the design of production system every effort is made that the real production system has these properties.

The model from Fig. 2 can be described using matrices to describe the T - timed PN.

$$N = (\boldsymbol{P}, \boldsymbol{T}, \boldsymbol{pre}, \boldsymbol{post}, \boldsymbol{M_0}, \boldsymbol{Tempo}) \tag{7}$$

where **P** = {p1, p2, ..., pm} is a finite set of places
T = {t1, t2, …, tn} is a finite set of transitions,
pre – a direct incidence matrix, pre: **P** x **T** → N,

post – a back incidence matrix, post: **P** x **T**→ N,
$\mathbf{M_0}$ - initial marking $\mathbf{M_0}$: **P** → N,
Tempo (ti) = di – time intervals indicate the implementation of transitions, where di ≥ 0.

From the matrix **pre** and **post,** it is possible to calculate the incidence matrix **C** by the relation (8).

$$\boldsymbol{C} = \boldsymbol{post} - \boldsymbol{pre} \tag{8}$$

The incidence matrix for the PN from Fig. 2 has the form (9).

$$\boldsymbol{C} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix} \tag{9}$$

4. Experimental verification of the above methodology

In this chapter methodology described in chapter 2 will be verified experimentally.

4.1. The analysis of state reachability in PN

The desired state of $\mathbf{M_x}$, which is used for this calculation, is a state in which it is necessary to transform the system. The state when a product is made is regarded as a desired state. By substituting values into the equation (2), it is possible to calculate the appropriate firing sequence. Equation (10) calculates the firing sequence needed to transform the PN to the desired state.

$$\boldsymbol{\sigma} = \boldsymbol{C}^{-1} * \begin{pmatrix} 4 & 5 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 + & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{10}$$

Firing sequence takes its values of non-negative whole numbers and then the desired state is reachable. If the desired state is not reachable, the firing sequence will not take the value of non-negative whole numbers. When the state of making one product is considered to be a desired state and post 1 is not ready, the desired state of $\mathbf{M_x}$ is changed. The new desired state is substituting into equation (2). Equation (11) calculates the firing sequence for the new desired state.

$$\boldsymbol{\sigma} = \boldsymbol{C}^{-1} * \begin{pmatrix} 4 & 5 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 + & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{bmatrix} 1,28 \\ 1,12 \\ 1,04 \\ 0,99 \\ 0,93 \\ 0,88 \\ 0,82 \\ 0,77 \\ 0,72 \end{bmatrix} \tag{11}$$

From the result of equation (11) it is clear that a calculated firing sequence is impossible, therefore the desired state is unreachable.

4.2. Analysis of the transition time in Petri net

For modeling production system from Fig.1 the matrix **Tempo** has the form (12). Times of operation in this case were found out experimentally. Since there is no real model the individual time intervals were measured in the PLC controller. Twenty measurements were carried out in total and for ease of calculation, these values were rounded to whole numbers. Designate times for operations in the design phase are difficult, which can not be determined experimentally, but based on the specifications of the technology and equipment it is possible to calculate these times.

$$\boldsymbol{Tempo} = [5 \quad 19 \quad 3 \quad 10 \quad 3 \quad 84 \quad 7 \quad 1 \quad 6] \tag{12}$$

Time transformation of PN from the initial to the desired state can be calculated by substituting values into equation (3). The time of transformation in this way can be calculated only for systems which manufacture only one product in one cycle. Calculation time of production for several products at the same time, the calculation can not be done with a simple equation. To calculate the transition time for more products a more complex calculation is needed, which is not devoted to this article. The production system is modeled on the PN from Fig. 2 is a transition time can be calculated using equation (13).

$$t = [5 \quad 19 \quad 3 \quad 10 \quad 3 \quad 84 \quad 7 \quad 1 \quad 6] * \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 138 \qquad (13)$$

From the result of equation (13) implies that the production of one product takes 138 seconds.

Substituting values into equation (4) and calculating can be determined the ideal hour performance.

$$3600/138 = 26,087 \qquad (14)$$

On the basis of calculating the ideal hourly performance, we can say that FPS can produce the maximum of 26 pieces of products per hour.

To calculate real hourly performance the relation (5) is used for 70% and 80% performance of production system. Therefore, real hourly performance is

a) with 80% performance

$$3600/t * (80/100) = 20,8696 \qquad (15)$$

b) with 70% performance

$$3600/t * (70/100) = 18,26 \qquad (16)$$

On the basis of calculating the real hourly performance, it is possible to say that FPS can produce 18 to 20 pieces of products per hour.

4.3. Analysis of the transformation cost in Petri net

Using a firing sequence, it is possible to determine the transformation cost of PN from the initial to the desired state, i.e. the production cost of one piece of a product. It is impossible to determine the cost to perform each operation in this case because it is a school model and not a real production system.

If we take the notional cost to perform each operation into account, these costs can be entered in the matrix (17).

$$\boldsymbol{F} = [10 \quad 5 \quad 10 \quad 15 \quad 5 \quad 30 \quad 5 \quad 20 \quad 10] \qquad (17)$$

Then it is possible to calculate the cost to transform the network into the desired state by using the equation (18).

$$k = [10 \quad 5 \quad 10 \quad 15 \quad 5 \quad 30 \quad 5 \quad 20 \quad 10] * \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 110 \qquad (18)$$

The cost to transformation the PN from the initial to the desired state after substituting values into equation (9) and calculating is 110, that is the notional cost to produce one product is 110 euros. This example is only illustrative to demonstrate other options of a firing sequence utilization.

5. Conclusion

The area of the design and implementation of production systems is nowadays becoming increasingly important. Major role in this respect have new future plants which seek to minimize human intervention in the production process. In the design of production systems, a careful analysis becomes increasingly important. One of the methods of analysis of production systems is the utilization of PN. Using the model of the production system, such properties as reachability state of the production process, production time, production costs or performance of production systems can be analyzed. By performing the analysis of this data for different states, production systems with known characteristics can be implemented.

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