OPTIMAL CONTROL DESIGN FOR LABORATORY HELICOPTER MODEL

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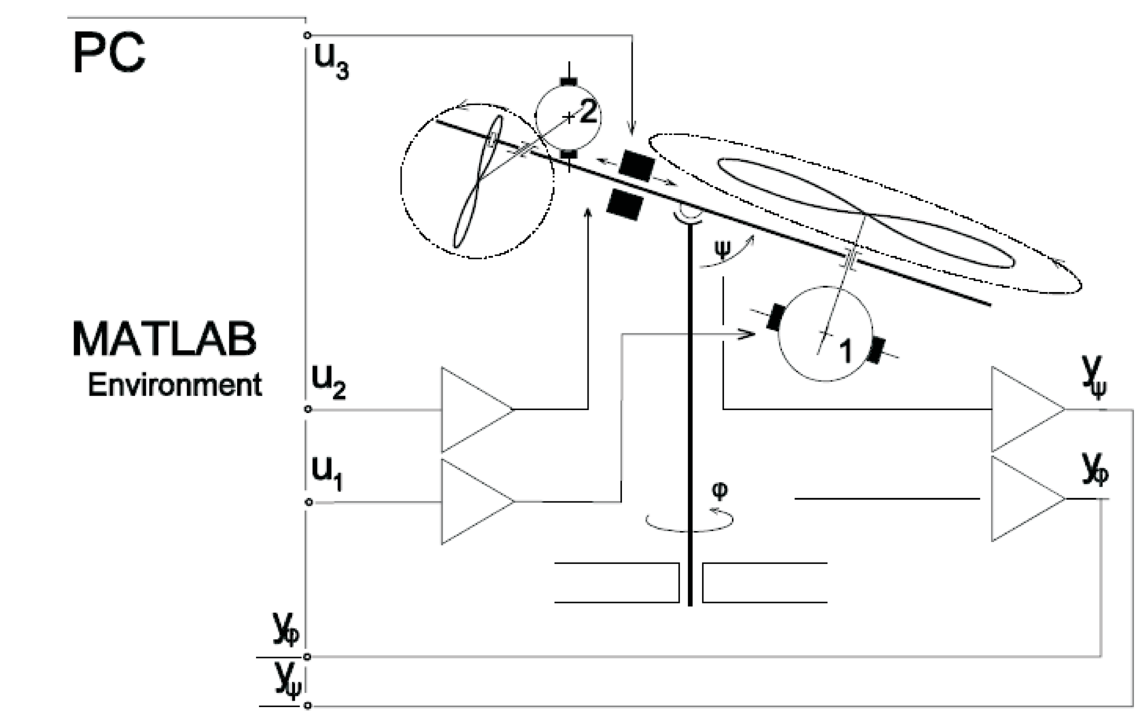
Mechanical system of helicopter model CE 150



Schematic diagram of the Helicopter Model

Mechanical system of helicopter

The mechanical system-helicopter has two degrees of the freedom, the rotation of the helicopter body with respect to the horizontal axis (an elevation y_1) and the rotation around the vertical axis (an azimuth y_2), which are measured by two sensors. The helicopter can move from (-130º,130º) in azimuth, and from (-45º,45º) in elevation. The inputs to the model are the voltages u_1 and u_2 affecting the main and the side rotor. Both the inputs are constrained between -1V and 1V.

Matlab/Simulink model

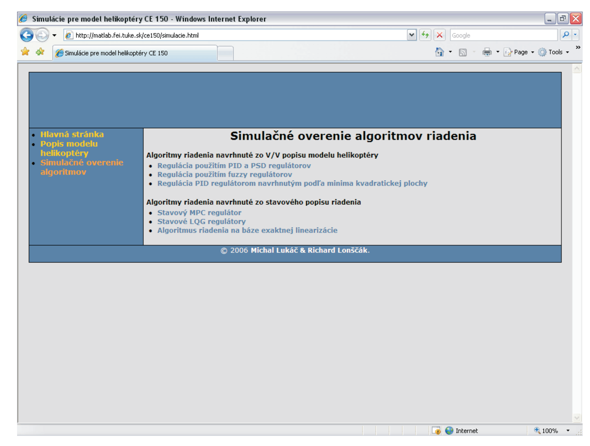
For verification of the predictive algorithm by designed control structure in the language Matlab/Simulink was created model of helicopter as S-function in the state space for an equilibrium point where as non-linear mathematical-physical model of the helicopter has as it's source.

Internet aplication

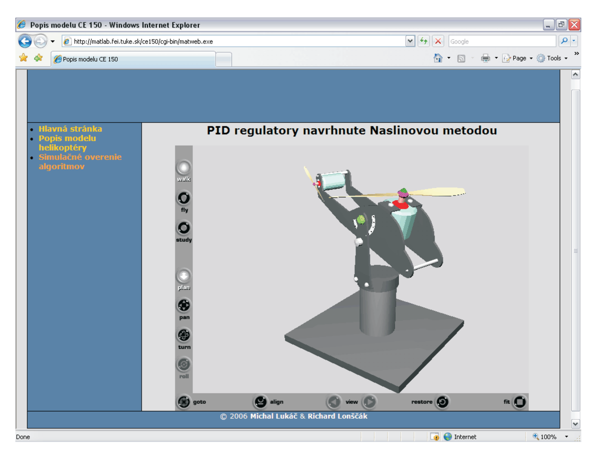
The analysis of the lab model-helicopter and the possibility of the optimal algorithms design by an Internet application is used for teaching subjects as Introduction to Nonlinear Systems, Theory of Optimal and Adaptive Systems, Control and Artificial Intelligence, which are lectured on the Department of Cybernetics and Artificial Intelligence and Faculty of Electrical Engineering and Informatics, Technical University of Košice. An Internet application enable to simplify the usage of designed MPC and modified LQG algorithms for the testing their properties at the dynamics of MIMO non-linear and unstable system.

Virtual laboratory

http://matlab.fei.tuke.sk/ce150/



Page of verification of algorithms



Virtual model of helicopter

State space predictive control algorithm

Model Predictive Control (MPC)

is the method of control in which the objective is to compute such a future control sequence, that the future system output is driven as close as possible the reference trajectory. This is accomplished by minimizing quadratic multistage cost function defined over the prediction horizon.

$$J_{MPC}(k)=\sum_{i=1}^{n}\sum_{j=0}^{N_p}\mu_i\left(\hat{y}_i(k+j\,|\,k)-r_i(k+j\,|\,k)\right)^2+\sum_{l=1}^{m}\sum_{j=0}^{N_p}\lambda_l u_l^2(k+j\,|\,k)$$

$$\boldsymbol{x}(k+1)=\boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{B}\boldsymbol{u}(k) \qquad \hat{\boldsymbol{y}}=\boldsymbol{G}\boldsymbol{u}+\boldsymbol{S}\boldsymbol{x}(k)$$

$$\boldsymbol{y}(k)=\boldsymbol{C}\boldsymbol{x}(k)+\boldsymbol{D}\boldsymbol{u}(k)$$

$$\hat{\boldsymbol{y}}=\begin{bmatrix}\hat{y}(k+1)\\ \hat{y}(k+2)\\ \vdots\\ \hat{y}(k+N_p)\end{bmatrix},\quad \boldsymbol{u}=\begin{bmatrix}\boldsymbol{u}(k)\\ \boldsymbol{u}(k+1)\\ \vdots\\ \boldsymbol{u}(k+N_p-1)\end{bmatrix}\quad \boldsymbol{G}=\begin{bmatrix}\boldsymbol{CB} & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ \boldsymbol{CAB} & \boldsymbol{CB} & \cdots & \boldsymbol{0}\\ \vdots & \vdots & \ddots & \boldsymbol{0}\\ \boldsymbol{CA}^{N_p-1}\boldsymbol{B} & \boldsymbol{CA}^{N_p-2}\boldsymbol{B} & \cdots & \boldsymbol{CB}\end{bmatrix},\quad \boldsymbol{S}=\begin{bmatrix}\boldsymbol{CA}\\ \boldsymbol{CA}^2\\ \vdots\\ \boldsymbol{CA}^{N_p}\end{bmatrix}$$

$$J_{MPC}=\left(\hat{\boldsymbol{y}}-\boldsymbol{r}\right)^T\boldsymbol{M}\left(\hat{\boldsymbol{y}}-\boldsymbol{r}\right)+\boldsymbol{u}^T\boldsymbol{L}\boldsymbol{u}$$

$$J_{MPC}=\frac{1}{2}\boldsymbol{u}^T\boldsymbol{H}\boldsymbol{u}+\boldsymbol{b}^T\boldsymbol{u}+f_0 \qquad \boldsymbol{H}=2\left(\boldsymbol{G}^T\boldsymbol{M}\boldsymbol{G}+\boldsymbol{L}\right)$$

$$\boldsymbol{b}^T=2\left(\boldsymbol{S}\boldsymbol{x}(k)-\boldsymbol{r}\right)^T\boldsymbol{G}\boldsymbol{M}$$

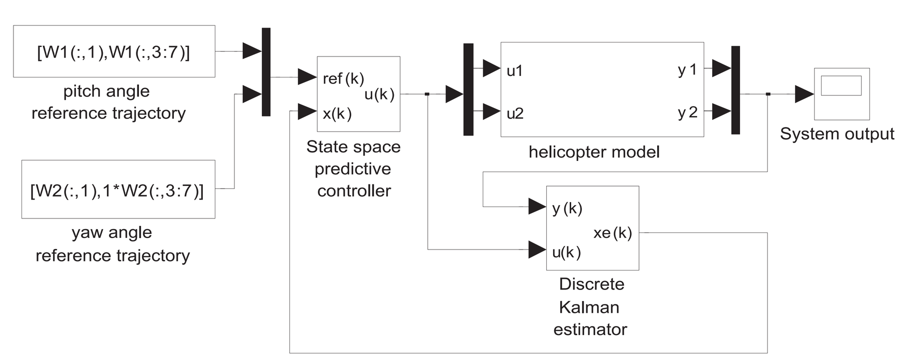
$$\boldsymbol{f}_0=\left(\boldsymbol{S}\boldsymbol{x}(k)-\boldsymbol{r}\right)^T\left(\boldsymbol{S}\boldsymbol{x}(k)-\boldsymbol{r}\right)$$

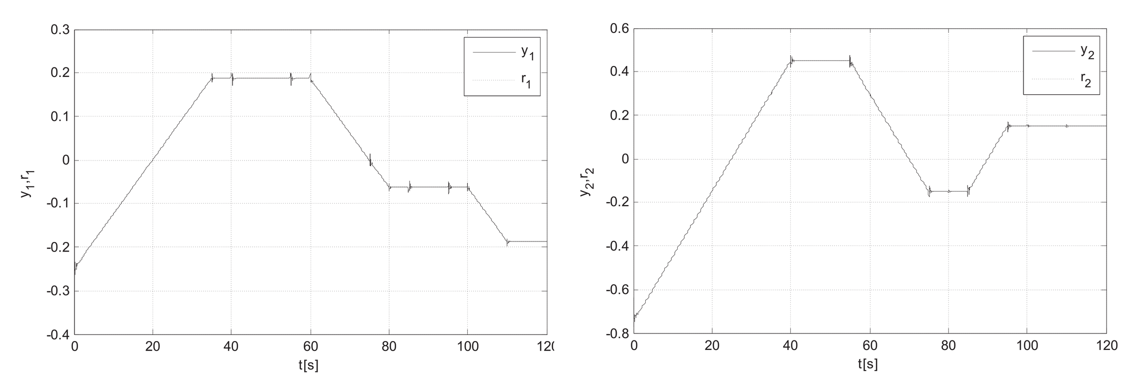
$$\boldsymbol{u}(k)=\boldsymbol{K}\left(\boldsymbol{r}-\boldsymbol{S}\boldsymbol{x}(k)\right)$$

where $\boldsymbol{K}$ is the first m-rows of matrix

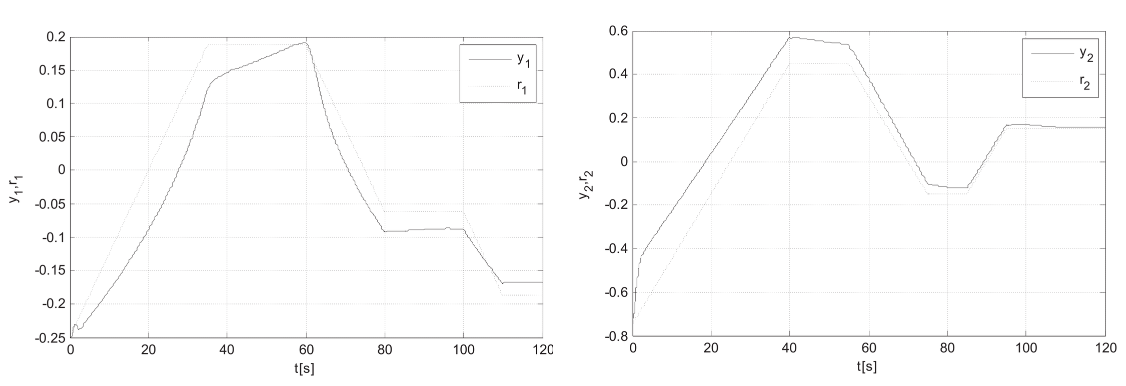
$$\left(\boldsymbol{G}^T\boldsymbol{M}\boldsymbol{G}+\boldsymbol{L}\right)^{-1}\boldsymbol{G}^T\boldsymbol{M}$$

Simulation verification of MPC algorithm





MPC - the tracking of the reference trajectories $r_1(k)$ and $r_2(k)$ by the outputs of the linear system $y_1(k)$ and $y_2(k)$



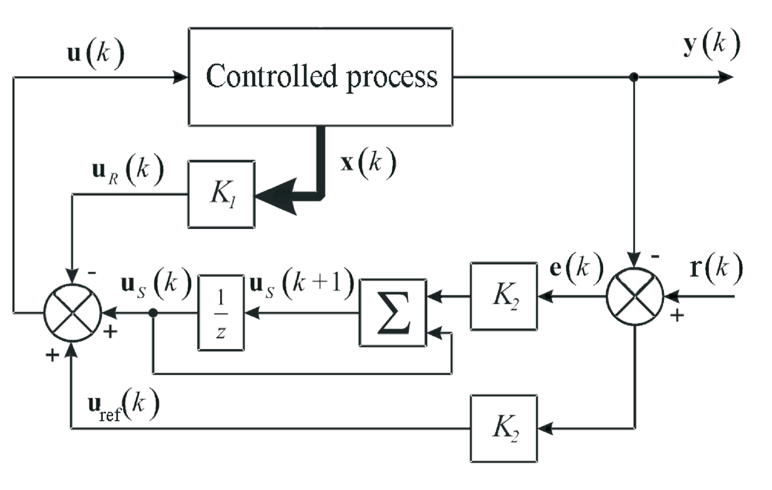
MPC - the tracking of the reference trajectories $r_1(k)$ and $r_2(k)$ by the outputs of the nonlinear system $y_1(k)$ and $y_2(k)$

Modification of LQ control algorithm

LQ control with integral action

The main idea of an LQ controller design is the minimization of a quadratic criterion which is weighting (diagonal matrix $\boldsymbol{R}$) the square of a manipulated variable and the square of a controlled variable (diagonal matrix $\boldsymbol{Q}$).

$$J\left(u(k)\right)=\sum_{k=1}^{N}\left[\boldsymbol{x}^T(k)\boldsymbol{Q}\boldsymbol{x}(k)+\boldsymbol{u}^T(k)\boldsymbol{R}\boldsymbol{u}(k)\right]+\boldsymbol{x}^T(N)\boldsymbol{P}\boldsymbol{x}(N)$$



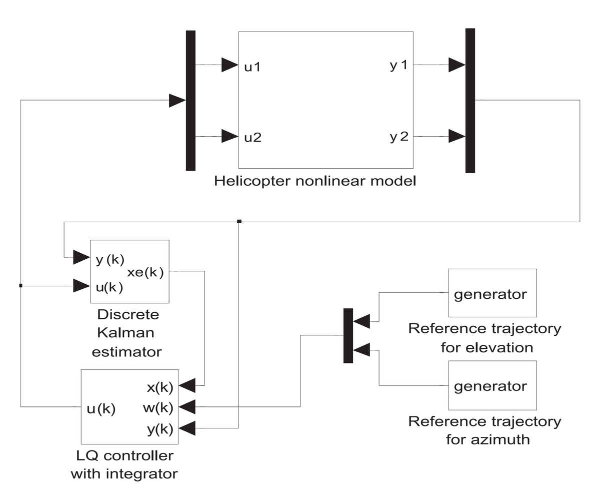
The state space LQ controller in feedback loop generates signal $u_R(k)=\boldsymbol{K}_1\boldsymbol{x}(k)$, where $\boldsymbol{K}_1$ in this point is non defined vector of the controller of the modified control scheme. The control input is created by three elements.

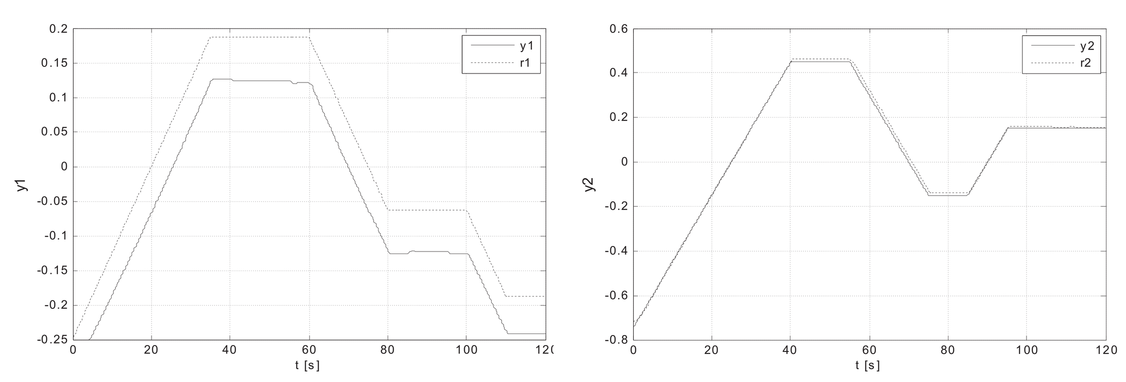
$$\boldsymbol{u}(k)=\boldsymbol{u}_R(k)+\boldsymbol{u}_S(k)+\boldsymbol{u}_{ref}(k)=-\boldsymbol{K}_1\boldsymbol{x}(k)+\boldsymbol{S}(k)+\boldsymbol{K}_2\boldsymbol{r}(k)$$

Implementation of agorithm to Matlab

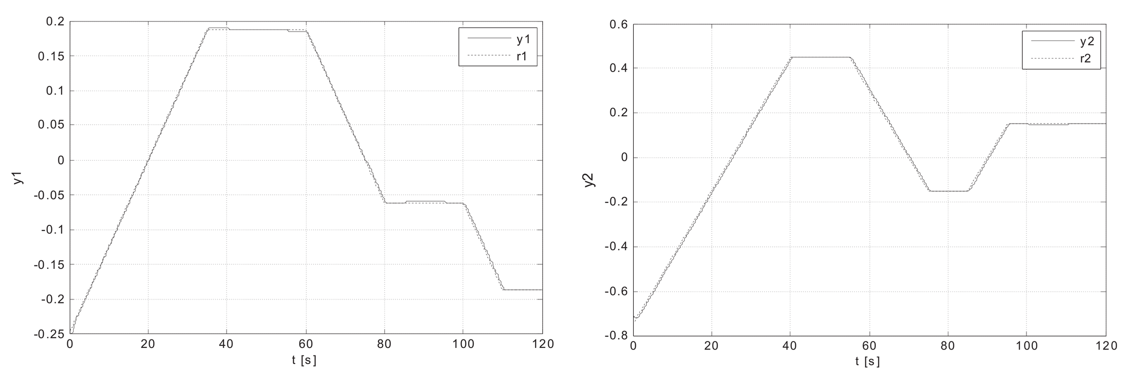
1. **Start** of simulation,
2. Calculation of the gain matrix $\boldsymbol{K}(k)$ in feedback loop by recursive enumeration of the Riccati equations in cycle, which will stop, if is executed condition $|\boldsymbol{K}(k)\ \boldsymbol{K}(k\text{-}1)|\leq\varepsilon$, where ε is the precision of enumeration,
3. **If** $t=t_{end\_simulation}$ **then** jump to step 11,
4. Calculation of vector of outputs $\boldsymbol{y}(k)$,
5. Load of the state vector $\boldsymbol{x}(k)$ an controlled outputs $\boldsymbol{y}(k)$ of the system,
6. Calculation of the gain matrixes $\boldsymbol{K}_1$, $\boldsymbol{K}_2$ from matrix $\boldsymbol{K}(k)$ in the feedback closed loop of the control system
7. Calculation of the control input $\boldsymbol{u}(k)=\boldsymbol{u}_s(k)\text{-}\boldsymbol{K}_1\boldsymbol{x}(k)+\boldsymbol{K}_2\boldsymbol{r}(k)$,
8. Calculation of the control input of summator $\boldsymbol{u}_s(k+1)=\boldsymbol{u}_s(k)+\boldsymbol{K}_2(\boldsymbol{r}(k)\text{-}\boldsymbol{y}(k))$,
9. Calculation of the state vector of the process $\boldsymbol{x}(k+1)=\boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{B}\boldsymbol{u}(k)+\boldsymbol{B}_d\boldsymbol{z}(k)$,
10. Calculation of matrix $\boldsymbol{K}(k+1)$ and jump to step 3,
11. **End** of simulation.

Simulation verification of LQ algorithm





LQ control - the tracking of the reference trajectories $r_1(k)$ and $r_2(k)$ by the outputs of the nonlinear system $y_1(k)$ and $y_2(k)$



LQ control with integral action - the tracking of the reference trajectories $r_1(k)$ and $r_2(k)$ by the outputs of the nonlinear system $y_1(k)$ and $y_2(k)$