# Explicit and Online Predictive Control of Nonlinear Hydraulic System

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***Abstract*—An explicit and online predictive control of nonlinear hydraulic system is presented in this paper. Obtained results of explicit predictive control are compared with results by the classical approach of predictive control, where an optimization task is executed in every sample of control closed loop. According to particular introduced algorithms program modules in simulation language Matlab are created and used in the simulation control of nonlinear hydraulic system. The main goal of this paper is to compare both approaches of predictive control of nonlinear dynamical systems based on a predictor in a linear form, summarize advantages and disadvantages of each approach.**

***Keywords*—nonlinear hydraulic system, state-space model based predictive control, explicit predictive control.**

## I. INTRODUCTION

The paper deals with the predictive control of nonlinear hydraulic system. Predictive control algorithms, which were used in the control are based on using a linear approximation of nonlinear model of controlled physical system. As controlled system the nonlinear simulation model of hydraulic system, which is located in the Laboratory of mechatronic systems at the Department of Cybernetics and Artificial Intelligence was used. The simulation of its control was carried out in Matlab/Simulink with created program modules in the simulation language of Matlab on the basis of theoretical background of predictive control algorithms.

As the hardware configuration and the mathematically-physical description of used hydraulic system were presented in [1], in this paper it is introduced very briefly in the part II. Next the basic principle of predictive control on the basis of predictor in a linear form, the theoretical derivation and the programming design of control algorithms are mentioned. The part III is devoted to predictive control algorithms, which is based on the state space model of controlled system. In the part IV the explicit solution of predictive control is presented. In the end of this paper results of nonlinear hydraulic system simulation control by mentioned algorithms are depicted and mutually compared.

## II. NONLINEAR HYDRAULIC SYSTEM

As it have already been mentioned in the introduction, the used nonlinear model of hydraulic system was introduced in [1], eventually in [2], where its principle of dynamics, the hardware configuration, the communication way and the control by digital PID algorithms running in Matlab on PC level were alleged. For that reason only physical structure and a systemic view of this model for control purpose is presented in this paper. A schematic illustration of hydraulic system is depicted in Fig. 1, whereby particular physical parameters are:

$S$ - intersection of tanks,
$S_{v1}$, $S_{v2}$ - intersection of outlets of both tanks,
$h_{max}$ - height of tanks (maximal liquid level).

Physical quantities shown in Fig. 1 are:

$f_m(t)$ - pump's motor frequency,
$h_1(t)$, $h_2(t)$ - current levels of liquid in both tanks.

Sensors, which scan the current liquid level in both tanks are marked as $S_{n1}$ and $S_{n2}$.

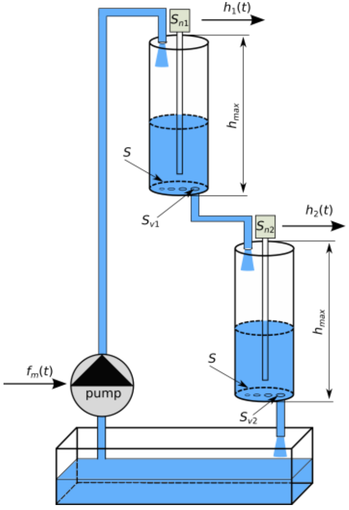

Fig. 1: The hydraulic system of two tanks

The systemic view of introduced hydraulic system is depicted in Fig.2, where besides already mentioned quantities, $q_{in1}(t)$ and $q_{in2}(t)$ is inflow to the first and the second tank.

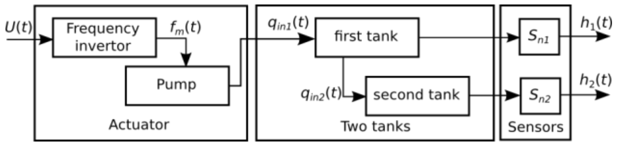

Fig. 2: The systemic view of hydraulic system

Regarding to the control process, a voltage of frequency invertor in range 0 – 10V constitutes the control action quantity and the liquid level in the second tank $h_2(t)$ is the controlled quantity. State quantities are liquid levels in both tanks $h_1(t)$ and $h_2(t)$, which have limit values 0 – 0.3m.

## III. ONLINE PREDICTIVE CONTROL

The basic principle of predictive control algorithms consists in using linear model to compute a prediction of future behaviour of controlled system on the length of prediction horizon (Fig. 3). Regarding to used linear model it is possible to divide the set of predictive control algorithms into two categories:

A. algorithms of **G**eneralized **P**redictive **C**ontrol (GPC), which use input/output description [3],
B. state **S**pace **M**odel-based **P**redictive **C**ontrol algorithms (SMPC).

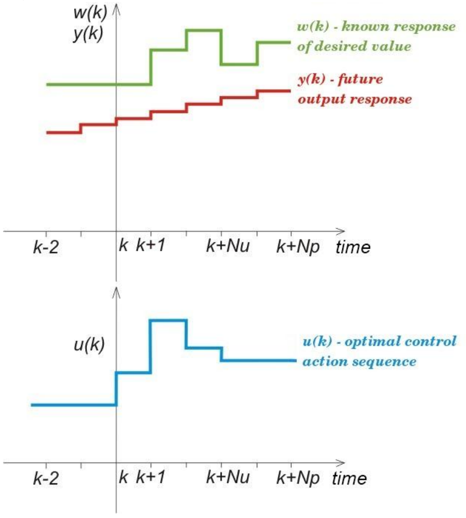

Fig. 3: Predictive control principle

The general control structure with predictive control algorithm is depicted in Fig. 3, where $\boldsymbol{w}$ is vector of output desired on the length of prediction horizon, $\boldsymbol{u}(k)$ is control action computed by GPC or SMPC algorithm, $\boldsymbol{d}(k)$ is disturbance vector and $\boldsymbol{y}(k)$, $\boldsymbol{x}(k)$ is output, state vector of controlled system, respectively.

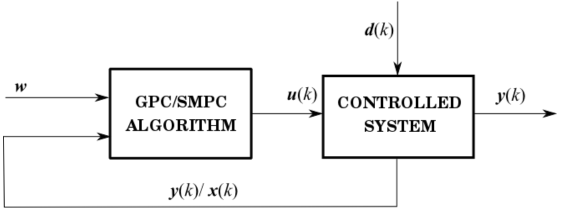

Fig. 4: The control structure with GPC/SMPC algorithm

Control action computing is based on a criteria function minimization $J_{MPC}$, where setting parameters are represented by weighing matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$. Thus, it is carried out by optimization task. Additionally, predictive control algorithms have a big advantage, that it is possible to transform control action computing to the quadratic programming task and involve physical quantities constraints of controlled system. The criteria function minimization with constraints by quadratic programming means more computing demands, because it is possible to carry out such calculation only by numerical way.

Control action computing by predictive control algorithms is carried out on the basis of receding horizon strategy. The optimization task is executed at each sample period and results to the optimal sequence of control action. However, only the first element of computed sequence is used as system control input. This procedure is repeated again in next sample instances.

Relatively, many modifications of basic predictive control principle exist. In this paper two algorithms of SMPC are mentioned.

In this part an online predictive control algorithm, which keeps the receding horizon principle is introduced. It uses a discrete state space description of dynamic system

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A_d}\boldsymbol{x}(k) + \boldsymbol{B_d}\boldsymbol{u}(k) \\ \boldsymbol{y}(k) &= \boldsymbol{C}\boldsymbol{x}(k) + \boldsymbol{D}\boldsymbol{u}(k) \end{aligned}, \tag{8}$$

where $\boldsymbol{x}(k)$, $\boldsymbol{u}(k)$ and $\boldsymbol{y}(k)$ is a vector of system's states, inputs and outputs, and matrices $\boldsymbol{A_d}$, $\boldsymbol{B_d}$, $\boldsymbol{C}$, $\boldsymbol{D}$ with particular dimensions contain coefficients represented system dynamics for predictor derivation.

According to [5] in the case of SMPC algorithm it is used criteria function

$$\begin{aligned} J_{\mathrm{MPC}} = &\sum_{i=N_1}^{N_p} \boldsymbol{Q}\left[\hat{\boldsymbol{y}}(k+i) - \boldsymbol{w}(k+i)\right]^2 + \\ &+ \sum_{i=1}^{N_u} \boldsymbol{R}\left[\Delta\boldsymbol{u}(k+i-1)\right]^2 \end{aligned}, \tag{9}$$

where $\boldsymbol{N_p}$ and $\boldsymbol{N_u}$ is predicion and control horizon, $\boldsymbol{Q}$ and $\boldsymbol{R}$ are weighing matrices and $\hat{\boldsymbol{y}}(k)$ is predicted output.

Regarding to the fact, that we want to weigh control action rate $\Delta\boldsymbol{u}(k)$ in the criteria function, we can isolate $\Delta\boldsymbol{u}(k)$ in predictor derivation according to [5]:

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}_d\boldsymbol{x}(k) + \boldsymbol{B}_d\boldsymbol{u}(k-1) + \boldsymbol{B}_d\Delta\boldsymbol{u}(k) \\ \boldsymbol{y}(k) &= \boldsymbol{C}\boldsymbol{x}(k) + \boldsymbol{D}\boldsymbol{u}(k) \\ \boldsymbol{u}(k) &= \boldsymbol{u}(k-1) + \Delta\boldsymbol{u}(k) \end{aligned} \tag{10}$$

According to [6] and provided that $\boldsymbol{D} = \boldsymbol{0}$ it is possible to derive the predictor by iteration of discrete state space description equations (10) step by step in the form

$$\hat{\boldsymbol{y}} = \boldsymbol{V}\boldsymbol{x}(k) + \boldsymbol{G}_1\boldsymbol{u}(k-1) + \boldsymbol{G}_2\Delta\boldsymbol{u} = \boldsymbol{y}_0 + \boldsymbol{G}_2\Delta\boldsymbol{u}\,, \tag{11}$$

where

$$\boldsymbol{G}_1 = \begin{pmatrix} \boldsymbol{C}\boldsymbol{B}_d \\ \boldsymbol{C}\left(\boldsymbol{A}_d + \boldsymbol{I}\right)\boldsymbol{B}_d \\ \vdots \\ \boldsymbol{C}\left(\boldsymbol{A}_d^{Np-1} + \cdots + \boldsymbol{A}_d + \boldsymbol{I}\right)\boldsymbol{B}_d \end{pmatrix}$$

$$\boldsymbol{G}_2 = \begin{pmatrix} \boldsymbol{C}\boldsymbol{B}_d & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{C}\left(\boldsymbol{A}_d^{Np} + \boldsymbol{I}\right)\boldsymbol{B}_d & \boldsymbol{C}\boldsymbol{B}_d & \ddots & \vdots \\ \vdots & \ddots & \ddots & \boldsymbol{0} \\ \boldsymbol{C}\left(\boldsymbol{A}_d^{Np-1} + \cdots + \boldsymbol{A}_d + \boldsymbol{I}\right)\boldsymbol{B}_d & \cdots & \boldsymbol{C}\left(\boldsymbol{A}_d + \boldsymbol{I}\right)\boldsymbol{B}_d & \boldsymbol{C}\boldsymbol{B}_d \end{pmatrix}. \tag{12}$$

After the predictor equation (11) substitution to the matrix form of criteria function $J_{MPC}$ (9)

$$J_{\mathrm{MPC}} = \left(\hat{\boldsymbol{y}} - \boldsymbol{w}\right)^T \boldsymbol{Q}\left(\hat{\boldsymbol{y}} - \boldsymbol{w}\right) + \Delta\boldsymbol{u}^T\boldsymbol{R}\Delta\boldsymbol{u} \tag{13}$$

and after multiplying it is possible to express it by the quadratic form

$$J_{MPC} = c + 2\boldsymbol{g}^T\Delta\boldsymbol{u} + \Delta\boldsymbol{u}^T\boldsymbol{H}\Delta\boldsymbol{u}, \tag{14}$$

where $\boldsymbol{g}^{\boldsymbol{T}}$ is gradient, $\boldsymbol{H}$ is Hessian and $c$ is a constant.

According to [4] an analytic formula for optimal control action on the length of control action $N_u$

$$\Delta \boldsymbol{u} = -\boldsymbol{H}^{-1}\boldsymbol{g} \qquad (15)$$

can be expressed on the basis of condition

$$\frac{\partial J_{GPC}}{\partial \Delta \boldsymbol{u}} \overset{!}{=} \boldsymbol{0}. \qquad (16)$$

In the case when constraints of physical quantities regarding is required in control action computing, it is neccessary to solve minimizaton of (14) by quadratic programming, for example with the *quadprog* function, which is part of *Optimization Toolbox* in Matlab.

The SMPC algorithm uses information about states $\boldsymbol{x}(k)$ of controlled system in the feedback branch of control structure. The block of control algorithm in Fig. 4 can be shown as Fig. 5, where the prediction of system free response $\boldsymbol{y}_f$ is computed from current values of states $\boldsymbol{x}(k)$.

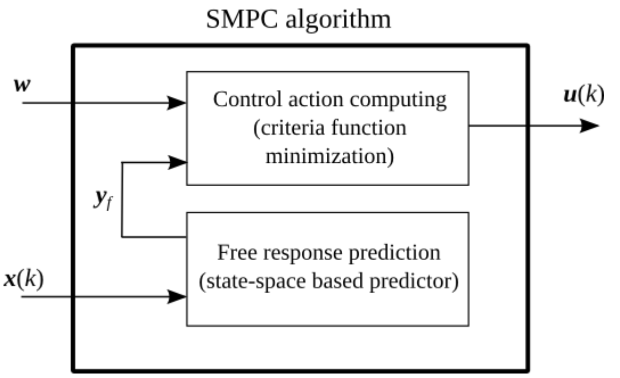

Fig. 5: The detail of SMPC algorithm block

## IV. EXPLICIT PREDICTIVE CONTROL

The main disadvantage of predictive control algorithms, which are based on the receding horizon strategy is high calculation demands to criteria function minimization with constraints regarding. This drawback can be eliminated by an explicit solution of optimal control action computing in advance, before control process. As the first, the scientific group led by prof. Morari from ETH in Zürich was concerned in the explicit solution of optimal control with constraints [7]. Besides many things, results of their research is the Multi-parametric toolbox (MPT), which contains functions for design, analysis and simulation of dynamic systems control by explicit predictive control on the basis of multiparametric programming [8].

According to [9] the result of multiparametric quadratic programming is explicitly computed control action $\boldsymbol{u}^*(k) = f(\boldsymbol{x}(k), \boldsymbol{w}(k))$ for possible values of states $\boldsymbol{x}(k)$, which can be used in the control structure in consequence.

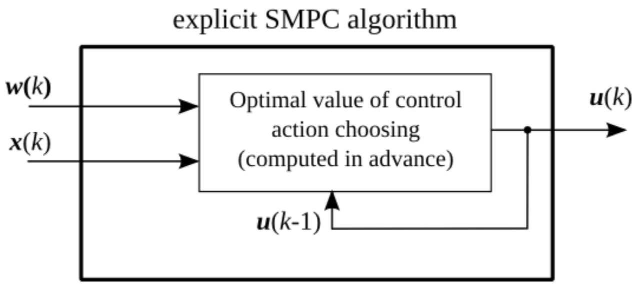

Fig. 6: The detail of explicit SMPC algorithm block

Forasmuch as the control algorithm selects an appropriate control action depending up to the current and desired state of controlled system in the frame of control process cycle, it is possible to use the structure depicted in Fig. 3 for dynamic system control. The detail of explicit predictive control algorithm block structure is shown in Fig. 6.

The basic algorithm, which makes possible to create a program of control structure for dynamic systems simulation control with MPT toolbox's functions is depicted in Fig. 8.

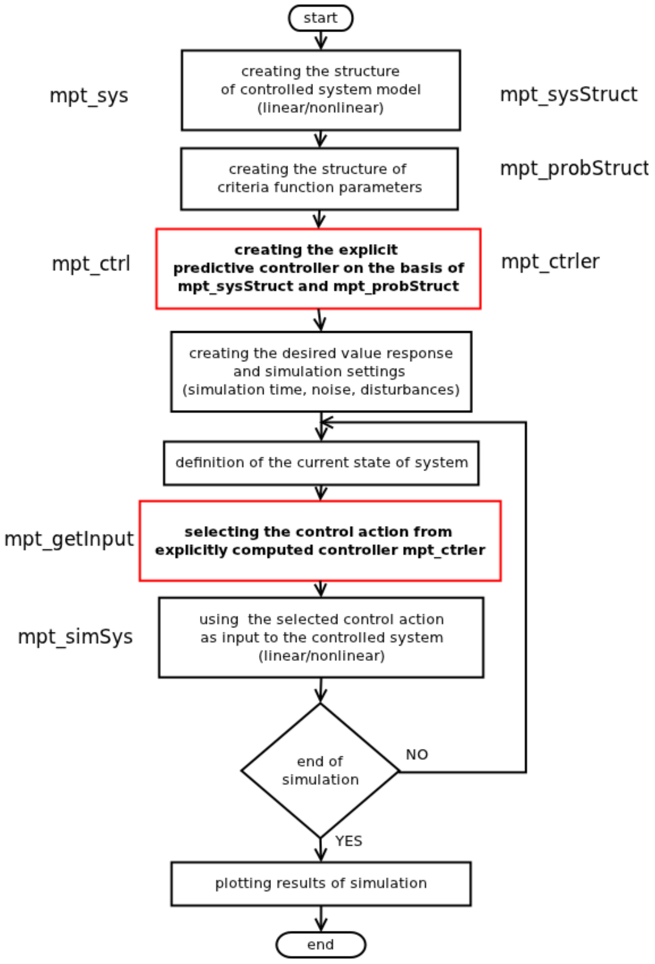

Fig. 7: The flow chart for programming the control structure with explicit predictive control algorithm

## V. CONTROL RESULTS COMPARISON

In this part results of nonlinear hydraulic system control with both mentioned algorithms are shown, whereby we used next settings of criteria function parameters: $\boldsymbol{N_p} = 10$, $\boldsymbol{N_u} = 2$, $\boldsymbol{Q} = 100\boldsymbol{I}$, $\boldsymbol{R} = 0.001\boldsymbol{I}$, sample period $T_{vz} = 1$s. In Fig. 8 results of control, where the desired value of liquid level in the second tank had constant value 0,13m is depicted. Because of fast liquid level onset in the first tank it was necessary to limit the input voltage to 8V, otherwise a liquid overflow in the first tank would happen. In Fig. 9 results of control with variable desired value are shown. While results of control to constant value are similar, in control, when the desired value is varying, a late reaction of the explicit predictive control algorithm is apparently noticeable. This kind of shift is caused by the fact that the MPT predictive control algorithm does not take the consideration to desired value time response on the length of prediction horizon, but only in the current sample instance.

Regarding to the speed of control action computation in each control process sample instance the explicit solution was almost three times faster then the online approach. An obtained average spending time of one control process cycle in simulation control on the same computer (CPU Intel i5-2410M 2,3GHz, RAM 4GB, Win7 64-bit) was: online – 5.2ms, explicit – 1.8s.

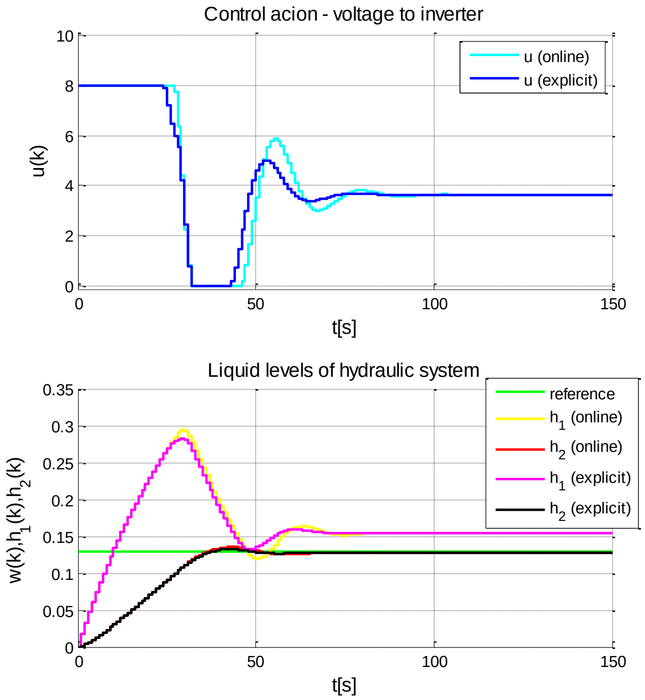

Fig. 8: Results of online and explicit predictive control of hydraulic system – constant desired value

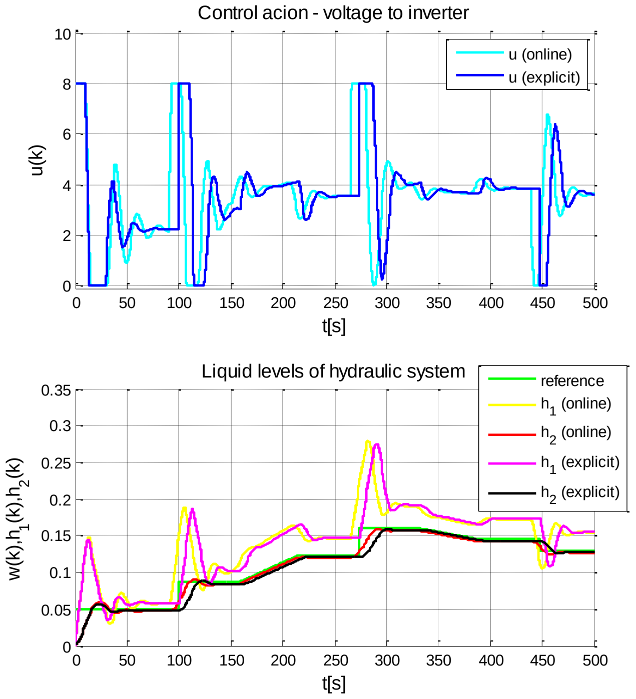

Fig. 9: Results of online and explicit predictive control of hydraulic system – variable desired value

## VI. CONCLUSION

It results from time responses depicted in the previous part that in the case when variable desired value is used it will be more preferable to use the online predictive control algorithm if fast achievement of desired value is required. However, in this case the computation demands of control algorithms should be taken into consideration. It is also necessary to adjust control requirements, for instance select suitable length of horizons, eventually pull out the sample time.

On the other side, in the case of physical system with fast dynamics control, when it is needed to use very short sample time, the explicit predictive control algorithm is more suitable then the online approach.

Both approaches of optimal control action computation are based on using the linear model of controlled physical system. However, using it in the explicit approach is more critical, especially when unmeasured disturbances may appear or in the case of control of systems, which dynamics is not possible to suitably approximate by linear model, whether for one or more operating points. This disadvantage is partly eliminated by control of piecewise systems, which is implemented enough in the frame of MPT toolbox, too.

## ACKNOWLEDGMENT

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