

# USING NEURAL NETWORKS FOR PHYSICAL SYSTEMS BEHAVIOUR PREDICTION

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## ABSTRACT

The paper is focused on modelling and behaviour prediction of physical systems by feedforward neural networks. Different structures for multilayer perceptron networks training on the basis of measured data of modelled physical system are mentioned in this paper. Neural networks trained as an input-output ARX and ARMAX model and a state-space model of dynamic system are used in a physical system behaviour prediction. A verification of successful neural networks training for prediction purpose is carried out by comparing with a simulation model of chosen physical system, which was created by analytic identification. Results of neural network output comparison with nonlinear simulation model are presented in the form of time responses. The main goal of this paper is to mention the possibility of using neural networks as physical systems dynamics predictors and thus avoid the necessity of mathematical model creation, e.g. in nonlinear differential equations form.

**Keywords:** ARX, ARMAX, feedforward neural networks, nonlinear physical system, behaviour prediction

## 1. INTRODUCTION

We are engaged in nonlinear dynamic system modelling by feedforward neural networks in this paper. The main goal is to mention the possibility of using the neural network as predictors of modelled system's dynamics and thus avoid the necessity of using the analytic identification.

However, for verification of training algorithms functionality we used the laboratory hydraulic system as modelled system, concretely its mathematical model represented by nonlinear differential equations. A derivation of mathematical model on the basis of physical principles was introduced in [1] and [2]. In this paper we target the training data obtaining from simulation hydraulic system model and in consequence the neural networks training as ARX, ARMAX and state-space model of nonlinear dynamic system according to [3].

After successful training we will use neural networks as modelled system behaviour predictor. Particular results we will compare with time responses obtained by numerical solution of nonlinear differential equations by Runge-Kutta method. In such a way we will check up the possibility to get the predictor of dynamic system on the basis of measured data without necessity to know exact mathematical model.

## 2. NONLINEAR MODEL OF HYDRAULIC SYSTEM

A schematic illustration of hydraulic system is depicted in Fig. 1, whereby particular physical parameters are:

- $S$  - intersection of tanks,
- $S_{v1}, S_{v2}$  - intersection of outlets of both tanks,
- $h_{max}$  - height of tanks (maximal liquid level).

Physical quantities shown in Fig. 1 are:

- $f_m(t)$  - pump's motor frequency,
- $h_1(t), h_2(t)$  - current levels of liquid in both tanks.

Sensors, which scan the current liquid level in both tanks are marked as  $S_{n1}$  and  $S_{n2}$ .

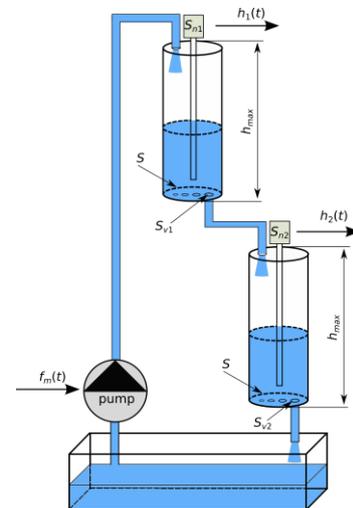


Fig. 1 The hydraulic system of two tanks

The systemic view of introduced hydraulic system is depicted in Fig.2, where besides already mentioned quantities,  $q_{in1}(t)$  and  $q_{in2}(t)$  is inflow to the first and the second tank.

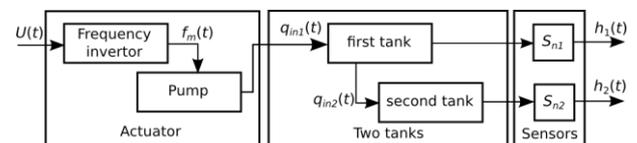


Fig. 2 The systemic view of hydraulic system

It is possible to derive nonlinear differential equations (1), which describe the hydraulic system dynamics by analytic identification on the basis of known physical principles:

$$\begin{aligned} \frac{dh_1(t)}{dt} & \frac{1}{S} k_p U(t) S_{v1} \sqrt{2gh_1(t)} \\ \frac{dh_2(t)}{dt} & \frac{1}{S} S_{v1} \sqrt{2gh_1(t)} S_{v2} \sqrt{2gh_2(t)} \end{aligned} \quad (1)$$

whereby  $g$  is the acceleration of gravity. The pump constant  $k_p$ , which describes a linear relationship between input voltage  $U(t)$  and inflow into the first tank  $q_{in1}(t)$ , was obtained by experimental measuring.

On the basis of nonlinear differential equations (1) we created a simulation model, which is depicted on Fig. 3. In our case this model constitutes the modelled system, whose outputs we will use for preparing training and testing data.

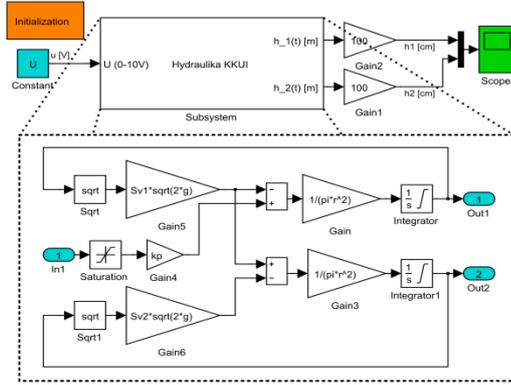


Fig. 3 Nonlinear simulation model of hydraulic system

### 3. USED NEURAL NETWORK MODELS

In this part three different approaches to feedforward neural networks training on the basis of measured data from modelled system are introduced. We are concerning the multilayer perceptron network (MLP) with one hidden layer, which structure is depicted on Fig. 4, whereby number of neurons on input and output layer is given by number of network's inputs and outputs. Number of neurons on hidden layer is an adjustable parameter [4].

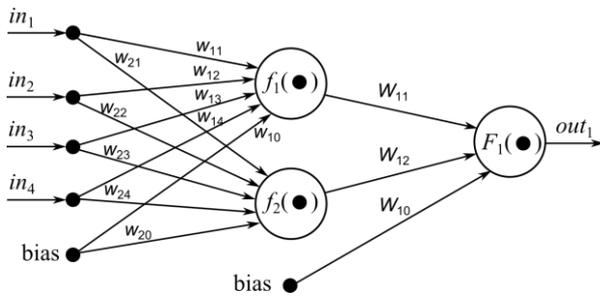


Fig. 4 MLP network: 4 inputs, 2 hidden units, 1 output

According to [3] the output value  $out_i$  can be computed by equation

$$out_i = F_i \left( \sum_{j=1}^{nh} W_{i,j} f_j \left( \sum_{k=1}^{ni} w_{j,k} in_k + w_{j,0} \right) + W_{i,0} \right) \quad (2)$$

where  $in_k$  is the  $k$ -th input,  $w_{j,k}$  is the weight between particular input and neuron on hidden layer,  $W_{i,j}$  is the weight between hidden layer output and output neuron and  $out_i$  is the  $i$ -th output. Symbols  $w_{j0}$  and  $W_{i0}$  are weights of external inputs, which are called *bias* and have value 1. Functions  $F_i$  and  $f_j$  constitute activation functions, in our case it is liner and hyperbolic tangent function.

According to [5] after integrating biases into vector of network's inputs  $in$  it is possible to express vector of network's outputs  $out$  in matrix form:

$$out = F\{W f(w in)\} \quad (3)$$

where  $w$  and  $W$  are matrices of weights and  $f$  and  $F$  are vector functions. Next we will present used structures of neural networks, which we trained on the basis of nonlinear hydraulic system model time responses.

#### 3.1. Neural ARX model

The input-output ARX model, which describes physical systems dynamics in linear form can be written as

$$A_z(z^{-1})y(k) = B_z(z^{-1})u(k) + e(k) \quad (4)$$

where  $A_z(z^{-1})$ ,  $B_z(z^{-1})$  are polynomials,  $u(k)$  is input,  $y(k)$  is output of dynamic system and  $e(k)$  is output error or measuring noise [6].

According to [3] the neural network, which shapes ARX model has structure, which is shown on Fig. 5.

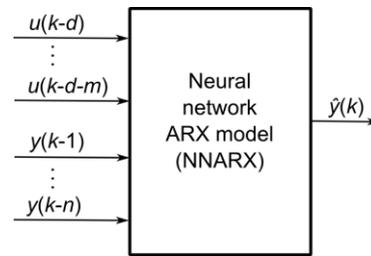


Fig. 5 Neural network of ARX model

Inputs of neural network as ARX model (NNARX) are constituted by values of modelled system input and output quantity in  $m$  and  $n$  previous samples, whereby  $d$  is system delay. In our case  $d = 0$ . Network's output is predicted value of system output  $\hat{y}(k)$ .

#### 3.2. Neural ARMAX model

The input-output ARMAX model has form

$$A_z(z^{-1})y(k) = B_z(z^{-1})u(k) + C_z(z^{-1})e(k) \quad (5)$$

where in addition to ARX model there is also polynomial  $C_z(z^{-1})$  [7].

On Fig. 6 structure of neural network, which shapes the ARMAX model according to [3] is shown. Opposite to the NNARX the NNARMAX model has inputs, which are given as a deviation between actual outputs and outputs computed by equation (3) in  $s$  previous samples.

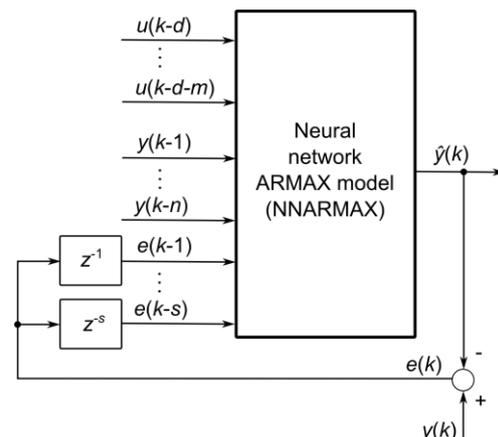


Fig. 6 Neural network of ARMAX model

### 3.3. Neural SSIF model

State-space model of physical systems has form

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}_d \mathbf{x}(k) + \mathbf{B}_d \mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C} \mathbf{x}(k) \end{aligned} \quad (6)$$

where  $\mathbf{A}_d$  is matrix of dynamics,  $\mathbf{B}_d$  is matrix of inputs,  $\mathbf{C}$  is matrix of outputs,  $\mathbf{x}(k)$  is vector of state quantities,  $\mathbf{u}(k)$  is vector of inputs and  $\mathbf{y}(k)$  is vector of system outputs. The structure of neural network representing the state-space innovations form model is shown on Fig. 7.

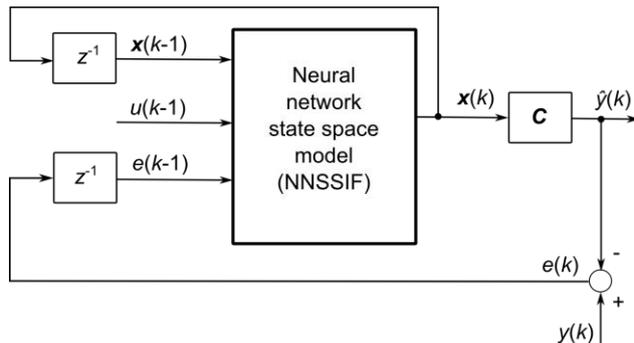


Fig. 7 Neural network of SSIF model

Unlike two previous neural models the NNSSIF model uses information about state quantities of modelled system.

### 4. TRAINING OF NEURAL NETWORKS

We carried out the training of introduced neural networks on the basis of the flow chart depicted on Fig. 8 by Neural Network Toolbox functions.

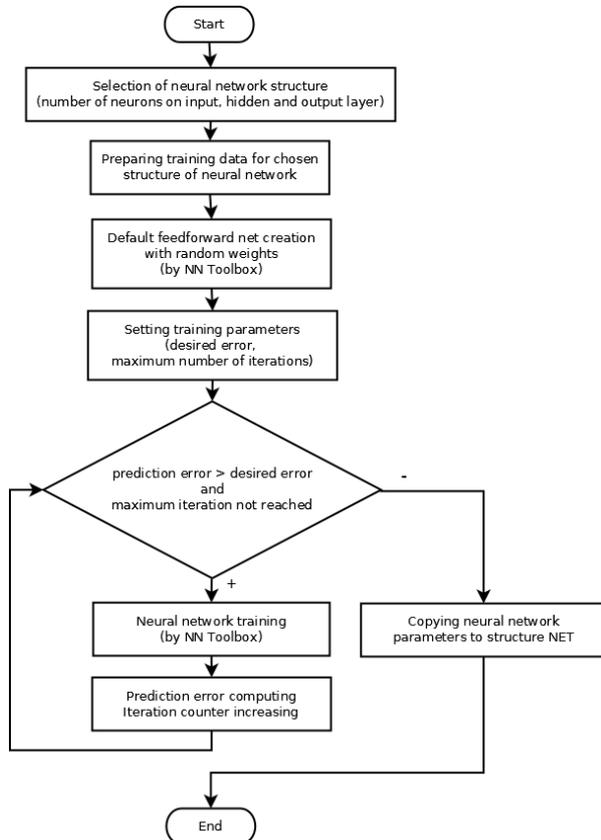


Fig. 8 Flow chart of neural network training

Neural networks structures, which we used in Hydraulic system modelling are written in Tab. 1. We obtained the best results with these settings for particular neural models.

Tab. 1 Structures of neural models

Number of	<i>NNARX</i>	<i>NNARMAX</i>	<i>NNSSIF</i>
<i>inputs</i>	6 $m=2, n=3$	9 $m=2, n=s=3$	4 2 elements in $\mathbf{x}$
<i>neurons on hidden layer</i>	10	20	5
<i>outputs</i>	1	1	2

As training data we used time responses obtained from the analytic model of Hydraulic system by numeric solving of nonlinear differential equations (1). Training data are depicted on Fig. 9.

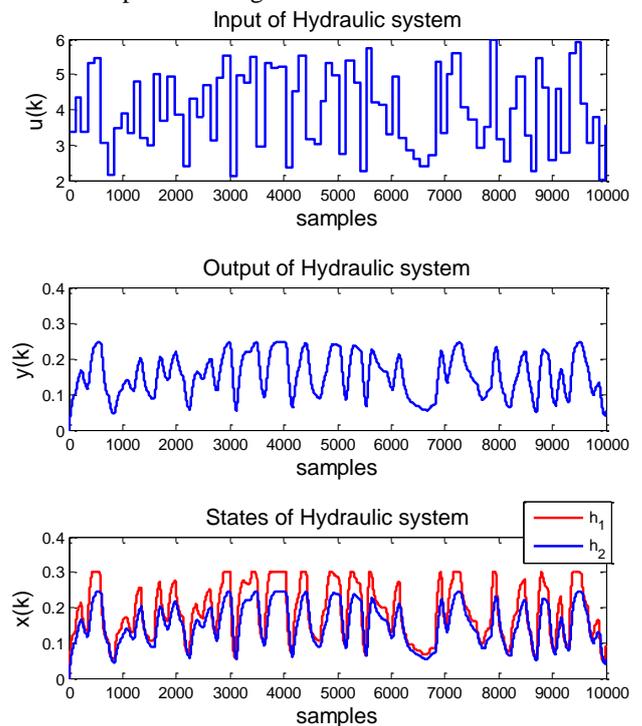


Fig. 9 Training data from Hydraulic system

The concrete values of weighing coefficients for particular neural model are result of training phase. Due to more flexibility and less complexity we designed structure, which we filled with necessary data about neural model obtained by Neural Network Toolbox, for example norming coefficients, weights between particular neurons and so on. We will use the structure *NET* in modelled system behaviour prediction computing.

### 5. VERIFICATION OF NEURAL NETWORKS

We carried out neural networks testing in two ways. Firstly, we applied another data obtained by simulation of nonlinear analytic model (Fig. 10) into neural network's inputs. We compared the neural network output with analytic model's output. Time responses of neural network output was the same as testing data. From that reason we don't present them in this paper. In such comparison we verified that neural network training was successful.

We verified the suitability of trained neural network for physical system behaviour prediction purpose in such way, that we applied testing data into the neural network inputs, which represent dynamic system input  $u(k)$  and for next network's inputs (representing the dynamic system output  $y(k)$  in  $n$  previous samples) we fed back neural network's output. Again, we compared obtained results with time responses computed on the basis of physical system's analytic model. This comparison is depicted on Fig. 11.

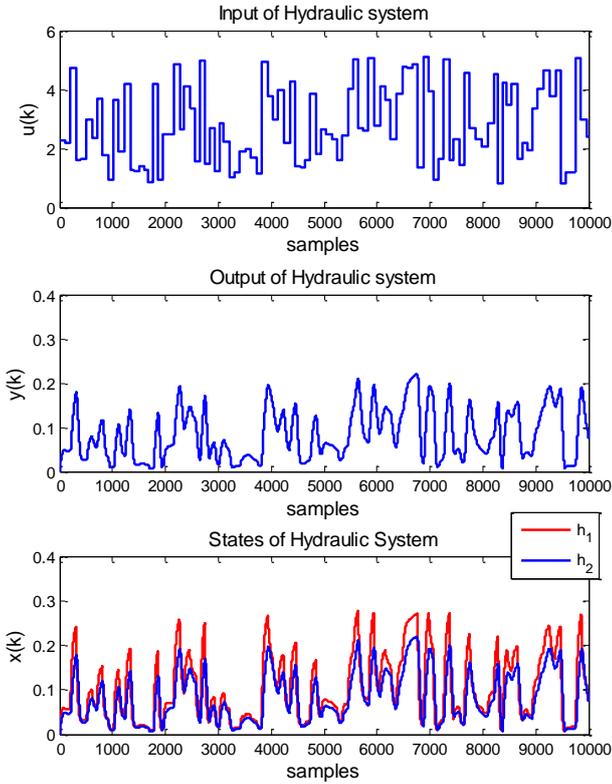


Fig. 10 Testing data from Hydraulic system

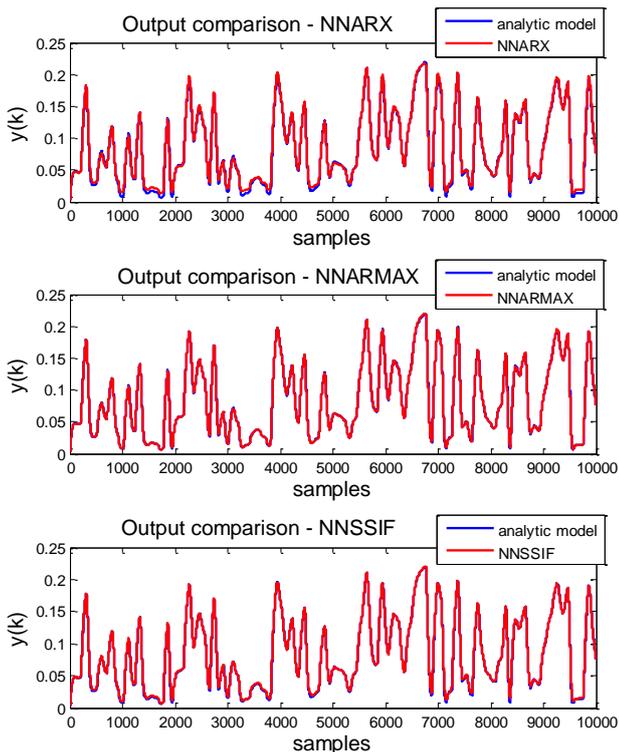


Fig. 11 Analytic model with neural network output comparison

It is clear from Fig. 11, that all of introduced neural models adequately approximate the dynamics of nonlinear Hydraulic system. However, the NNARX model returns a gentle deviation at the edge of system operating area.

The neural models outputs comparison with output of nonlinear analytic model of Hydraulic system is depicted on Fig. 12. The constant value  $u_c = 4V$  was used as input. The NNSSIF model seems to be the best choice for nonlinear systems approximating or behaviour prediction. We suppose that is induced by the fact, that time responses of state quantities constitute training data instead of system outputs. At the same time, the NNSSIF model has only 5 neurons on hidden layer, so the training of this neural model is faster than training of NNARMAX model, which was not able to model the dynamics of Hydraulic system adequately with the same structure.

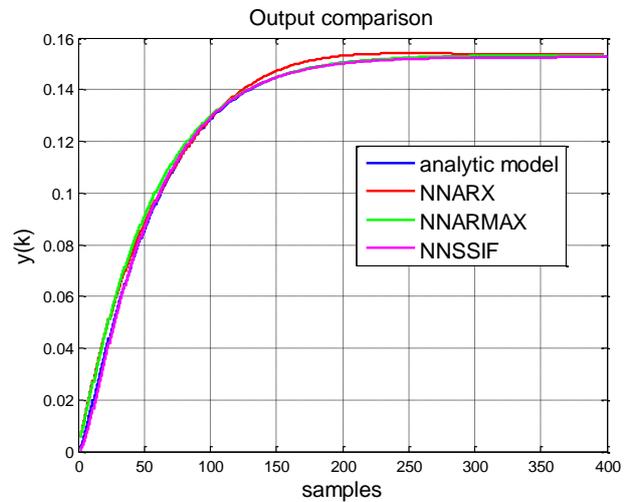


Fig. 12 Analytic model with neural network output comparison – constant input

## 6. CONCLUSIONS

We introduced the possibility to train feedforward neural networks for physical systems dynamics prediction in this paper. On the basis of results presented in part 5 we can conclude that neural networks with one hidden layer are really suitable for approximating the nonlinear systems dynamics and they can be used as predictors of physical systems behaviour. NNARMAX and NNSSIF models are more suitable, which is clear from neural networks outputs comparison with analytic model's output. It is caused mainly by their structure, forasmuch as the prediction error is applied as neural network's input during training. The advantage of NNSSIF model is also that it allows to predict state quantities values of modelled system.

We want to use every mentioned neural model as controlled system predictor and implement it to predictive control algorithms, whether on the base of input-output or state-space descriptions. In the frame of predictive control algorithms we want to use these neural models for controlled system free response prediction computing. We expect better control process, mainly because neural models are able to predict a nonlinear character of controlled systems better than classical ARX, CARIMA or state-space models.

## ACKNOWLEDGMENTS

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## BIOGRAPHIES

**Anna Jadlovská** was born October 29th, 1960. She received her MSc. degree in the field of Technical Cybernetics at the Faculty of Electrical Engineering of the Technical University in Košice in 1984. She defended her PhD thesis in the domain of Automatization and Control in 2001 at the same University; her thesis title was “Modelling and Control of Non-linear Processes Using Neural Networks”. Since 1993 she worked in the Department of Cybernetics and Artificial Intelligence Faculty of Electrical Engineering and Informatics Technical University in Košice as an Associate Assistant and since 2004 she has been working as an Associate Professor. Her main research activities include the problems of adaptive and optimal control – in particular predictive control with constraints for non-linear processes using neural networks and methods of artificial intelligence (Intelligent Control Design). She is the author of scientific articles and contributions to various journals and international conference proceedings, as well as being the co-author of some monographs.

**Štefan Jajčišin** was born August 22th, 1986. In 2010 he graduated (MSc.) with distinction at the Department of Cybernetics and Artificial Intelligence of the Faculty of Electrical Engineering and Informatics at Technical University in Košice. Since September 2010 he has been internal PhD. student at the Department of Cybernetics and Artificial Intelligence. The topic of his dissertation thesis is focused on predictive control algorithms for nonlinear dynamic systems. In addition, the sphere of his interest is physical dynamic system modeling, optimal control algorithms with constraints design and their verification on simulation and real laboratory models by Matlab/Simulink.