SAMI 2017—15th IEEE International Symposium on Applied Machine Intelligence and Informatics

Modelling and Control of a Cyber-Physical System represented by Hydraulic Coupled Tanks

D. Voščeк, A. Jadlovská
Technical University of Košice,
Faculty of Electrical Engineering and Informatics,
Department of Cybernetics and Artificial Intelligence, Košice, Slovakia

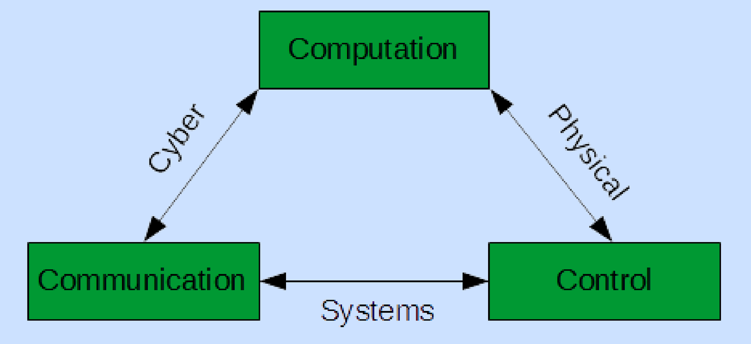
Abstract – This paper deals with hybrid modelling and control of cyber-physical systems, which are part of a recently introduced phenomenon Industry 4.0. As a case study we look at a hydraulic system with hybrid dynamics represented by coupled tanks. The dynamical system contains two discrete modes, specifically with and without interaction. The nonlinear hybrid system was modelled using s-functions and then its linear description was created in the discrete PWA representation with the help of HYSDEL modelling framework. Both s-functions and HYSDEL are part of MATLAB/Simulink environment. After validation of the discrete PWA representation in comparison with the nonlinear hybrid system, LQR synthesis with reference trajectory tracking was designed for both discrete modes. Subsequently optimal control designed on the linear hybrid system in the discrete PWA representation was verified on the nonlinear hybrid system.

Keywords— cyber-physical system, hybrid system, discrete hybrid automata, coupled tanks, optimal control

CPS overview

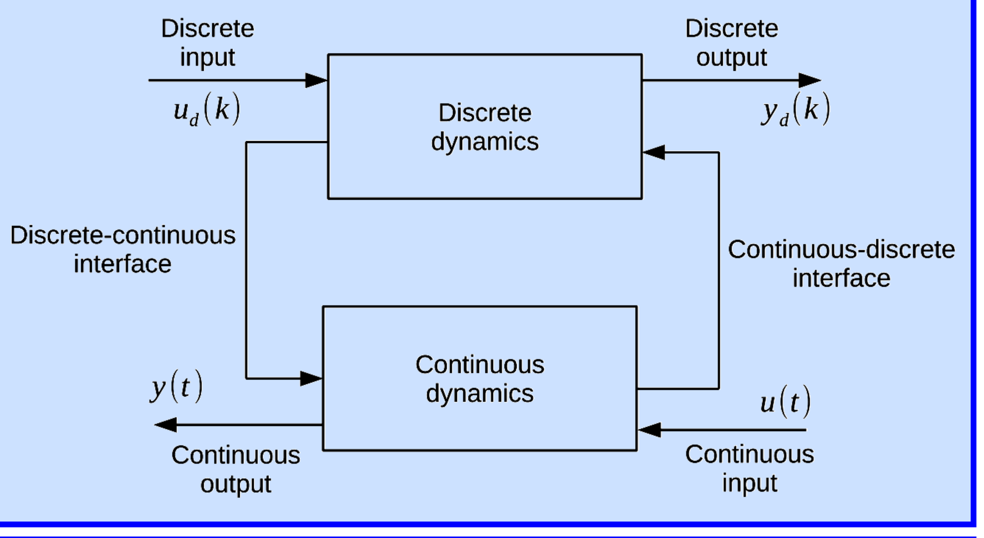
What is a Cyber Physical System?

- tight integration of computational and physical resources



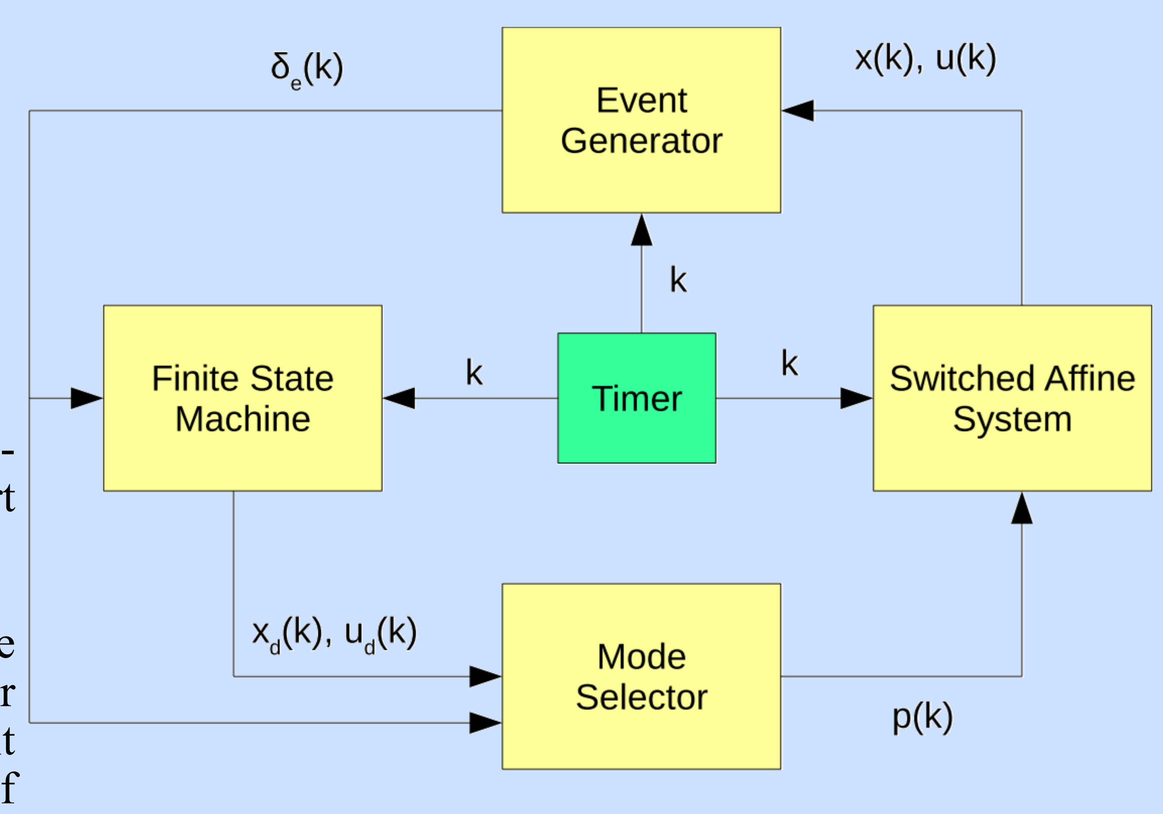
CPS challenges

- CPS composition
- Security and safety
- **Hybrid modelling and control**
- Architecture
- Sensor and mobile networks



Discrete Hybrid Automata

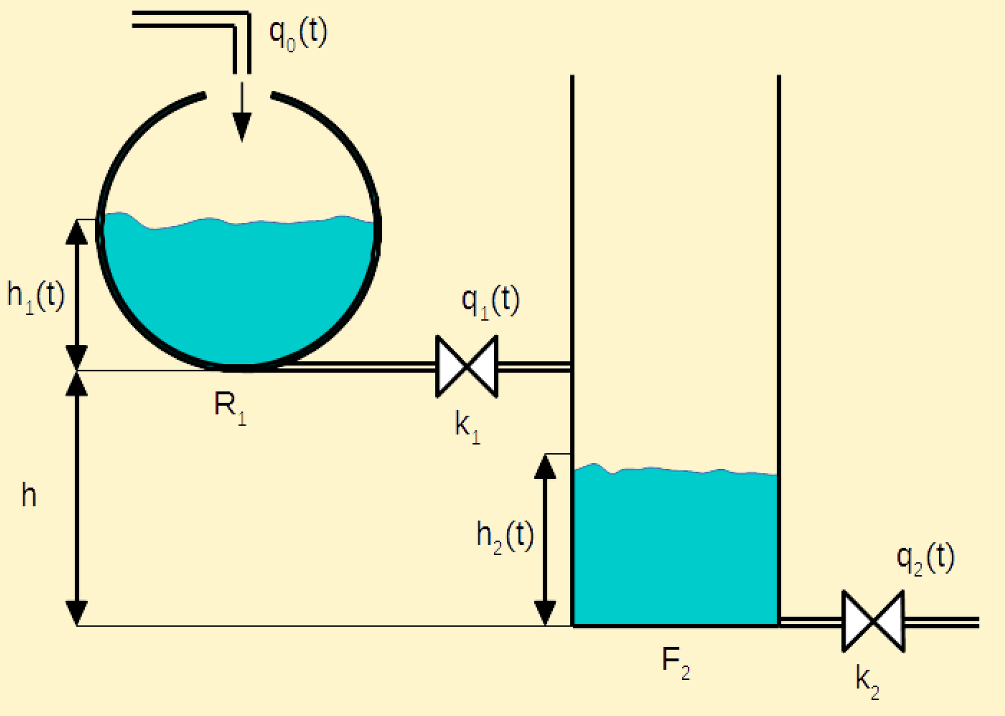
- **switched affine systems** - describe the continuous part of the hybrid system
- **event generator** - based on the fulfilment of boundary conditions provides a generation of a specific event
- **finite state machines** - represent the discrete part of the hybrid systems
- **mode selector** - based on the output of the event generator and finite state machine it defines the continuous part of the dynamical system



Hydraulic Coupled Tanks System

System parameters:

$h_1(t)$ - liquid level of the first tank
$h_2(t)$ - liquid level of the second tank
$q_0(t)$ - inflow
h - height of the first tank
R_1 - radius of the sphere tank
$F_1(t)$ - cross sectional area of the first tank
F_2 - cross sectional area of the second tank
k_1, k_2- flow resistance constants



Two discrete mods:

discrete mode **without interaction**:

$$\dot{h}_1(t) = \frac{q_0(t)}{F_1(t)} - \frac{k_1\sqrt{h_1(t)}}{F_1(t)},$$
$$\dot{h}_2(t) = \frac{k_1\sqrt{h_1(t)}}{F_2} - \frac{k_2\sqrt{h_2(t)}}{F_2}$$

discrete mode **with interaction**:

$$\dot{h}_1(t) = \frac{q_0(t)}{F_1(t)} - \frac{sign(L(t))k_1\sqrt{|L(t)|}}{F_1(t)}$$
$$\dot{h}_2(t) = \frac{\text{sign}(L(t))k_1\sqrt{|L(t)|}}{F_2} - \frac{k_2\sqrt{h_2(t)}}{F_2}$$
where $L(t) = h_1(t) - (h_2(t) - h).$

Linear approximation of the nonlinear model sampled with the time period T

discrete mode **without interaction**:

$$\Delta h_1(k+T) = \left(1 - \frac{k_{11}T}{F_{11}^s}\right)\Delta h_1(k) + \frac{\Delta q_0(k)T}{F_{11}^s}$$
$$\Delta h_2(k+T) = \frac{k_{11}T}{F_2}\Delta h_1(k) + \left(1 - \frac{k_{21}T}{F_2}\right)\Delta h_2(k)$$

discrete mode **with interaction**:

$$\Delta h_1(k+T) = \left(1 - \frac{k_{12}T}{F_{12}^s}\right)\Delta h_1(k) + \frac{k_{12}T}{F_{12}^s}\Delta h_2(k) + \frac{\Delta q_0(k)T}{F_{12}^s}$$
$$\Delta h_2(k+T) = \frac{k_{12}}{\frac{F_1^s}{T}}\Delta h_1(k) - \left(\frac{k_{12}}{\frac{F_2}{T}} + \frac{k_{22}}{\frac{F_2}{T}} - 1\right)\Delta h_2(k)$$

Parameters of the linear representation:

- k_{11}, k_{12}, k_{21}, k_{22} - modified flow resistances constants
- F_{11}, F_{12} - crosssectional areas of the first tank in its steady state for the specified discrete mode
- T - sample rate

PWA Representation of the Hydraulic Coupled Tanks System

Optimal control design for hybrid systems requires linear representation of the nonlinear system in the PWA (SAS) form

Switched affine systems:

$$\mathbf{x}(k+1) = \mathbf{F}_i\mathbf{x}(k) + \mathbf{G}_i\mathbf{u}(k) + \mathbf{f}_i, \quad i = 1,2$$
$$\mathbf{y}(k) = \mathbf{C}_i\mathbf{x}(k) + \mathbf{D}_i\mathbf{u}(k) + \mathbf{g}_i$$

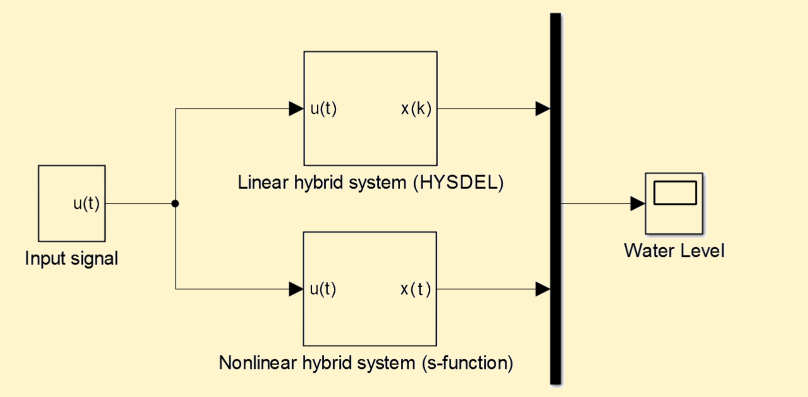
Discrete PWA form for the discrete mode **without the interaction**

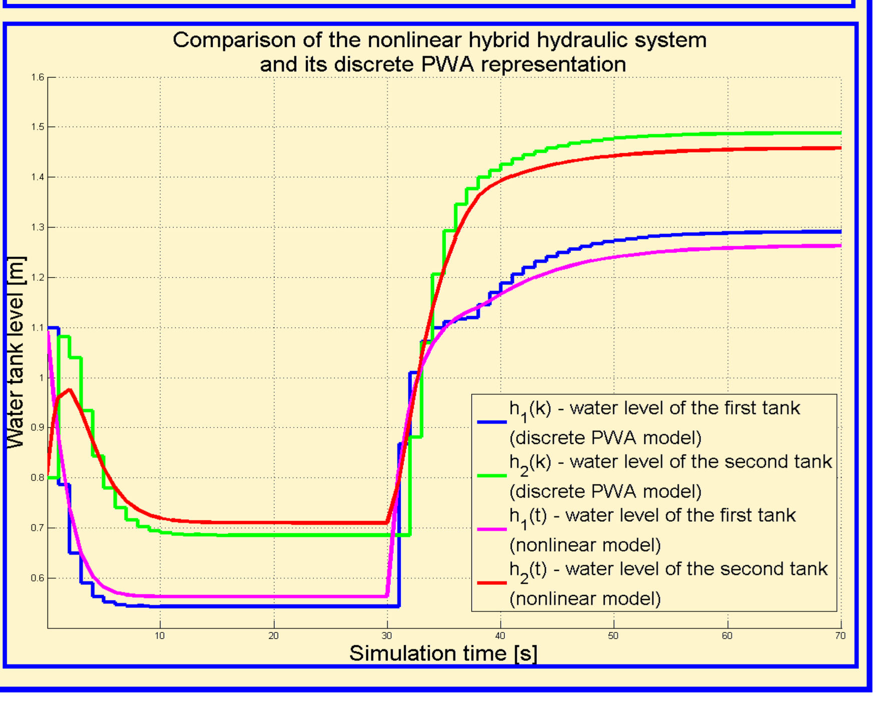
$$\mathbf{F}_1 = \begin{bmatrix} 1 - \frac{k_{11}T}{F_{11}^s} & 0 \\ \frac{k_{11}T}{F_2} & 1 - \frac{k_{21}T}{F_2} \end{bmatrix}, \mathbf{G}_1 = \begin{bmatrix} \frac{T}{F_{11}^s} \\ 0 \end{bmatrix}$$
$$\mathbf{C}_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \mathbf{f}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, g_1 = 0$$

Discrete PWA form for the discrete mode **with the interaction**

$$\mathbf{F}_2 = \begin{bmatrix} 1 - \frac{k_{12}T}{F_{12}^s} & \frac{k_{12}T}{F_{12}^s} \\ \frac{k_{12}T}{F_2} & 1 - \frac{k_{12}T}{F_2} - \frac{k_{22}T}{F_2} \end{bmatrix}, \mathbf{G}_2 = \begin{bmatrix} \frac{T}{F_{12}^s} \\ 0 \end{bmatrix}$$
$$\mathbf{C}_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \mathbf{f}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, g_2 = 0$$

Validation scheme for the discrete PWA representation of the nonlinear hybrid system





Optimal Control of the Hydraulic Coupled Tanks System

Optimal control is based on minimizing cost function:

$$J_i(u) = \sum_{n=1}^{\infty}\left(\Delta\mathbf{x}^T(k)\mathbf{Q}_i\Delta\mathbf{x}(k) + \Delta u_{fb}^T(k)\mathbf{R}_i\Delta u_{fb}(k)\right)$$

which solution is state-feedback gain $\mathbf{k}_i$

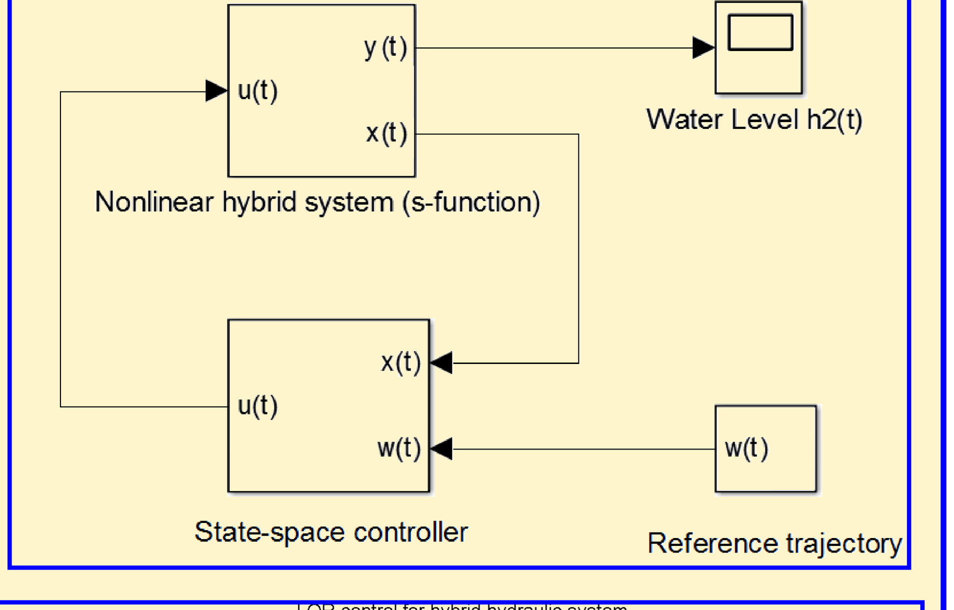
N_i - feed-forward gain

$$N_i = \frac{1}{\mathbf{C}_i(\mathbf{I} - (\mathbf{F}_i - \mathbf{G}_i\mathbf{k}_i))^{-1}\mathbf{G}_i}, i = 1,2$$

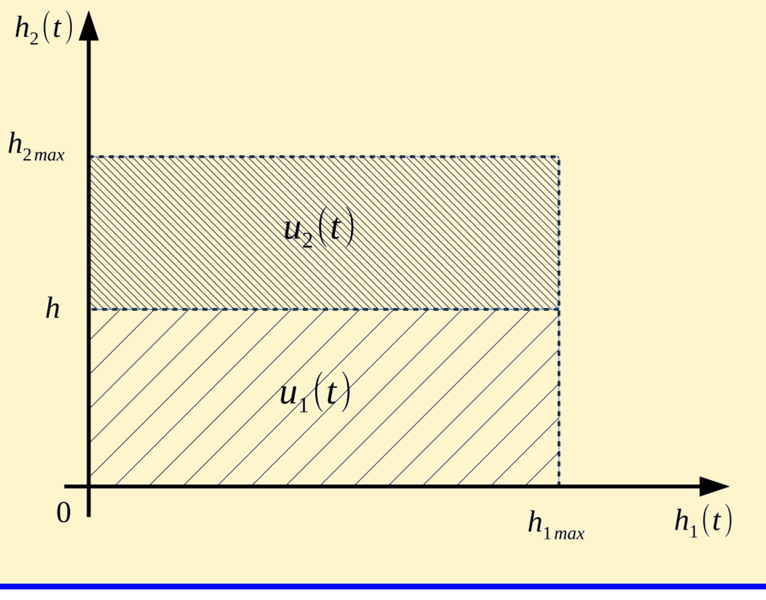
Control input signal:

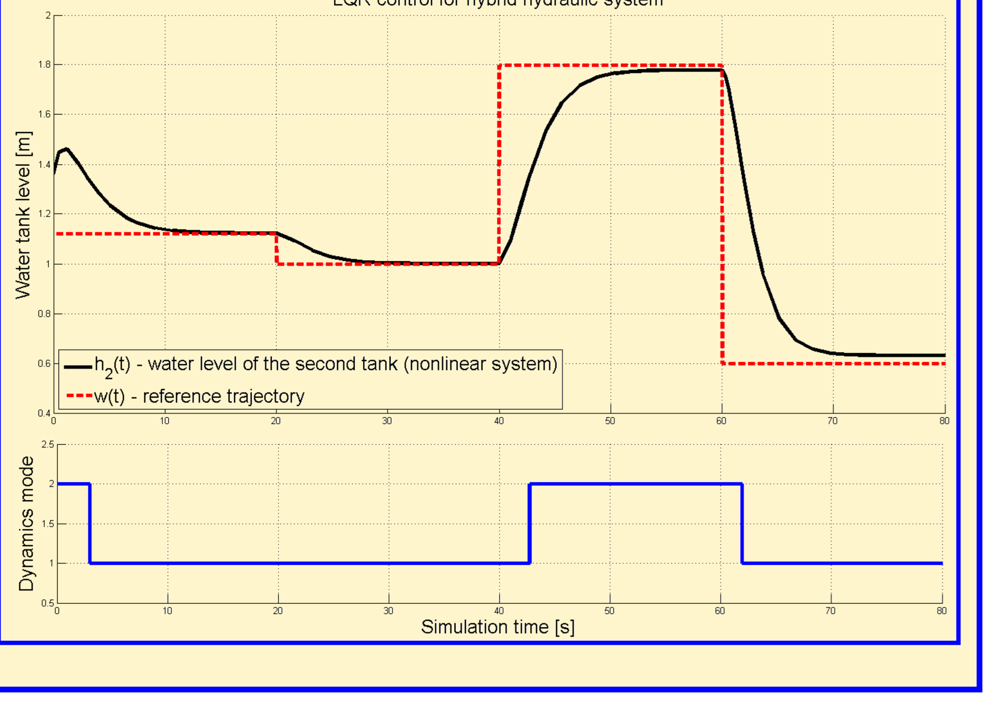
$$\Delta u(k) = -\mathbf{k}_i\Delta\mathbf{x}(k) + N_i\Delta w(k)$$

Control scheme for discrete LQR control with feedforward gain



State-space partitioning





ACKNOWLEDGEMENT: This work has been supported by the Research and Development Operational Program for project: Innovation Applications Supported by Knowledge Technology, ITMS:26220220182, co-financed by the ERDF (70%) and project KEGA - 001TUKE-4/2015 (30%).