SAMI 2017

IEEE 15th International Symposium on Applied Machine Intelligence and Informatics

PROCEEDINGS

January 26–28, 2017

Herl'any, Slovakia

IEEE Catalog Number: CFP1708E-USB
ISBN: 978-1-5090-5654-5

COMMITTEES

Honorary Chair

Lotfi A. Zadeh, USA

Honorary Committee

Ljiljana Trajkovic, Simon Fraser University, Canada
Stanislav Kmet, Technical University of Košice, Slovakia
Mihály Réger, Óbuda University, Budapest, Hungary
Liberios Vokorokos, Technical University of Košice, Slovakia
Anton Čižmár, Technical University of Košice, Slovakia

General Chairs

Imre J. Rudas, Óbuda University, Budapest, Hungary
Peter Sinčák, Technical University of Košice, Slovakia

International Organizing Committee Co-Chairs

Marián Bucko, Elfa, Slovakia
József Gáti, Óbuda University, Budapest, Hungary
Ladislav Főző, Technical University of Košice, Slovakia

Technical Program Committee Chairs

Szilveszter Kovács, University of Miskolc, Hungary
Jan Paralič, Technical University of Košice, Slovakia
Ivana Budinska, Slovak Academy of Science, Slovakia

Secretary General

Anikó Szakál
Óbuda University, Budapest, Hungary
E-mail: szakal@uni-obuda.hu

Iveta Zamecnikova
Technical University of Košice, Slovakia
E-mail: zamecnikova@elfa.sk

Technical Program Committee

Rudolf Andoga, Technical University of Košice, Slovakia
František Babič, Technical University of Kosice, Slovakia
Péter Baranyi, SZTAKI, Hungary
Peter Bednár, Technical University of Kosice, Slovakia
Balázs Benyó, Széchenyi István University, Győr, Hungary
Manuelle Bonacorossi, Scuola Superiore Santana, Italy
Marek Bundzel, Technical University of Košice, Slovakia
György Eigner, Óbuda University, Budapest, Hungary
Tamás Ferenci, Óbuda University, Budapest, Hungary
Ladislav Főző, Technical University of Košice, Slovakia
Róbert Fullér, Óbuda University, Budapest, Hungary
Alena Galajdová, Technical University of Košice, Slovakia
Tamás Haidegger, Óbuda University, Budapest, Hungary
Mikuláš Hajduk, Technical University of Kosice, Slovakia
László Horváth, Óbuda University, Budapest, Hungary
Ján Jadlovský, Technical University of Košice, Slovakia
Rudolf Jakša, Technical University of Košice, Slovakia
Aleš Janota, University of Žilina, Slovakia
Csaba Johanyák, Kecskemét College, Hungary
Levente Kovács, Óbuda University, Budapest, Hungary
Dušan Krokavec, Technical University of Košice, Slovakia
Marian Mach, Technical University of Košice, Slovakia
Kristína Machová, Technical University of Košice, Slovakia
Vladimír Modrák, Technical University of Košice, Slovakia
Igor Mokris, SAV Bratislava, Slovakia
Marek Penhaker, VSB Ostrava, Czech Republic
Martin Sarnovský, Technical University of Košice, Slovakia
Johanna Sápi, Óbuda University, Budapest, Hungary
Salvadore Sessa, Waseda University, Japan
Juraj Špalek, University of Žilina, Slovakia
Ágnes Szeghegyi, Óbuda University, Budapest, Hungary
Sándor Szénási, Óbuda University, Budapest, Hungary
Árpád Takács, Óbuda University, Budapest, Hungary
Márta Takács, Óbuda University, Budapest, Hungary
József K. Tar, Óbuda University, Budapest, Hungary
József Tick, Óbuda University, Budapest, Hungary
Kaori Yoshida, Kyushu Institute of Technology, Japan
Zoltán Vámossy, Óbuda University, Budapest, Hungary
Annamária R. Várkonyi-Kóczy, Óbuda University, Budapest, Hungary
Jan Vaščák, Technical University of Košice, Slovakia
Teréz Várkonyi, Óbuda University, Budapest, Hungary
Mária Vircíková, Technical University of Košice, Slovakia
Jozef Živčák, Technical University of Košice, Slovakia
Iveta Zolotova, Technical University of Košice, Slovakia

FOREWORD

Computational Intelligence and Intelligent Technologies are very important tools in building intelligent systems with various degree of autonomous behavior. These groups of tools support such features as ability to learn and adaptability of the intelligent systems in various types of environments and situations. The current and future Information Society is expecting to be implemented with the framework of the Ambient Intelligence (AmI) approach into technologies and everyday life. These accomplishments provide the wide range of application potentials for Machine Intelligence tools to support the AmI concept implementation. The number of studies indicates that this approach is inevitable and will play essential and central role in the development of Information Society in close future.

The essential importance of the Machine Intelligence in this historically challenging effort points out the responsibility of MI community including all fields like Brian-like research and applications, fuzzy logic, neural networks, evolutionary computation, multi-agent systems, artificial life, Expert Systems, Symbolic approaches based on logic reasoning, Knowledge discovery, mining, replication and many other related fields supporting the development and creation of the Intelligent System. The importance embedding these systems in various kinds of technologies should bring profit and different role of mankind in production and in everyday life. We expect to have intelligent technologies, solution and even humanoid robots to help the mankind to improve and keep the ideas of humanity and democracy.

The role of Machine Intelligence Quotient will play an important role in the future to be able to evaluate the degree of the autonomous behavior of the designed system. It is belief that it will be domain oriented problem and should also be important to use this information for decisions made by humans e.g. in evaluation of many information system in commercial tender to pick up the system with the highest MIQ. The usefulness of this parameter will be dependent on many influences including technological, domain oriented and also commercial aspects of the CI application in various systems. The commercial need to have “intelligent” solution and products should increase the interest for MI tools.

This year number of contribution showed up from mechanical Engineering domain, control and also pure computer science. We do believe that this multidisciplinarity will be very useful to emerge more AI applications in Information Society and will help making products and solutions more “intelligent”.

This proceedings is a small contribution of knowledge dissemination and presentation of important problems and advances in Computational intelligence theory and applications. Hungary and Slovakia as members of EU will do their best to contribute to European Research Area and support the development of Computational Intelligence technology for the benefit of the mankind.

Imre J. Rudas and Peter Sinčák
General Chairs

Table of Contents

Modelling and Control of a Cyber-Physical System represented by Hydraulic Coupled Tanks

Dominik Voššček*, Anna Jadlovská**
Technical University of Košice, Faculty of Electrical Engineering and Informatics,
Department of Cybernetics and Artificial Intelligence, Košice, Slovakia
*dominik.voscek@tuke.sk, **anna.jadlovska@tuke.sk

***Abstract*—This paper deals with hybrid modelling and control of cyber-physical systems, which are part of a recently introduced phenomenon Industry 4.0. As a case study we look at a hydraulic system with hybrid dynamics represented by coupled tanks. The dynamical system contains two discrete modes, specifically with and without interaction. The nonlinear hybrid system was modelled using *s-functions* and then its linear description was created in the discrete PWA representation with the help of HYSDEL modelling framework. Both *s-functions* and HYSDEL are part of MATLAB/Simulink environment. After validation of the discrete PWA representation in comparison with the nonlinear hybrid system, LQR synthesis with reference trajectory tracking was designed for both discrete modes. Subsequently optimal control designed on the linear hybrid system in the discrete PWA representation was verified on the nonlinear hybrid system.**

***Index Terms*—cyber-physical system, hybrid system, discrete hybrid automata, coupled tanks, optimal control.**

I. Introduction

Cyber-physical systems (CPS) integrate physical processes with computation and provide abstractions, modelling, analysis and design facilities for the implementation of the integrated whole [7]. CPS require networking and computing technologies to embrace not just the the physical dynamics but also the information, see Fig.1. The linkage among computing, network, control and physical systems require new design technologies [15].

One of the CPS challenges introduced in [14] is to model such a CPS as a hybrid system in a suitable hybrid modelling framework. This challenge lies in the center of the parts *Computation*, *Control* and *Communication* within Fig.1.

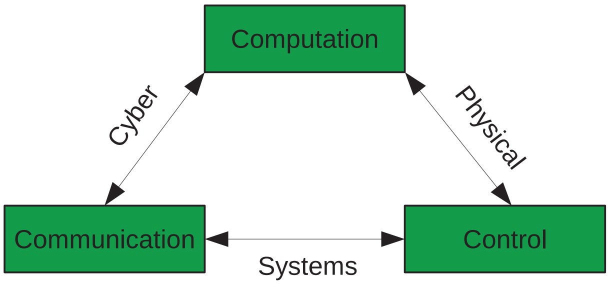

Fig. 1. Overview of a cyber-physical system [4]

Discrete hybrid automata (DHA) represents the most basic modelling frame for the hybrid systems [17]. This is mainly naturally caused by modelling the systems based on the physical laws (continuous part) and logic (discrete part). However, such a representation is not suitable for the hybrid systems analysis and control synthesis. Therefore, more appropriate mathematical frameworks were developed, e.g. discrete piecewise affine systems (dPWA). Both of the mentioned modelling frameworks (DHA and dPWA) will be introduced and presented within modelling and control of a hybrid hydraulic dynamical system, which was modelled but not controlled in [6]. Application of the LQ optimal control on the hybrid systems was utilized in [3] with the help of a dynamic optimization. Using different approach, we design optimal control for hybrid hydraulic system utilizing HYSDEL modelling framework in cooperation with discrete LQR control design.

II. Modelling Framework for Hybrid Systems

Hybrid system can be defined as a dynamical control system where the plant or the controller contains discrete modes that together with continuous equations govern the behaviour of the system [11]. Graphical representation of this definition is shown in Fig. 2. The scheme shows that the hybrid dynamical system contains continuous as well as discrete dynamics, inputs and outputs. The control of such systems involves discrete sets of observed data which define alternative interpretations of the system, mathematical models for each of these alternatives and optimal decisive strategy.

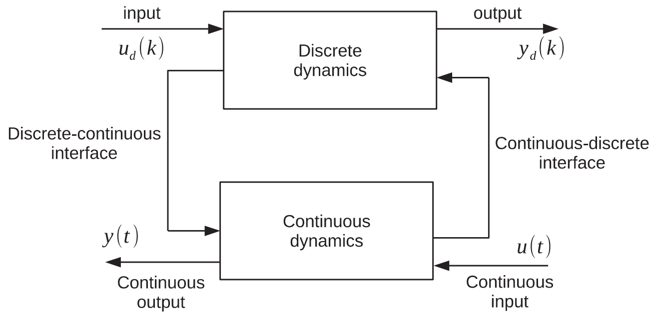

Fig. 2. Graphical representation of the hybrid dynamical control system

CPS belong to a hybrid systems category with the hybrid time domain [9]. The most significant representations of the hybrid systems are:

- discrete hybrid automata [17]
- piece-wise affine systems [2]
- mixed-logical dynamical (MLD) systems [1]
- switching control [10]

DHA is one of the most general hybrid systems description, where if the discrete part preserves its state then the continuous part of the dynamical system evolves according to the difference equations assigned to that discrete state [16]. The change of the system dynamics occurs due to an event, an impact or after fulfilling transition conditions in the state-space. This system dynamics change leads to the jump of the discrete state and change in the continuous part as well [16]. Modelling framework of DHA is composed of four parts [14], [16]:

- switched affine systems (SAS)
- event generator (EG)
- finite state machines (FSM)
- mode selector (MS),

which are depicted in Fig.3.

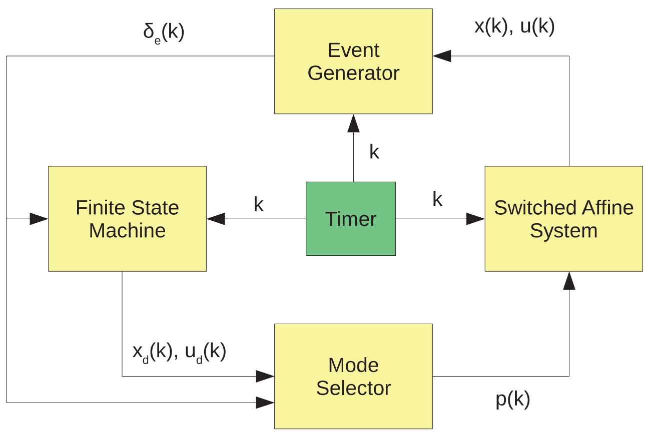

Fig. 3. The DHA framework for modelling CPS [16]

These parts of the DHA are defined as:

- **switched affine systems** - describe the continuous part of the hybrid system, which is defined as:

$$\begin{cases} x(k+1) = A_p x(k) + B_p u(k) + f_p \\ y(k) = C_p x(k) + D_p u(k) + g_p \end{cases}, \tag{1}$$

where $x(k)$ - state of the system, $u(k)$ - system input, $y(k)$ - system output, $\{A_p, B_p, C_p, D_p\}$ is a set of dynamical system matrices and vectors $\{f_p, g_p\}$ represent state and input independent parameters for discrete state $p(k)$.

- **event generator** - based on the fulfilment of boundary conditions it provides a generation of a specific event. Mathematically event generator can be described as:

$$\delta_e(k) = f_h(x(k), u(k), k) \tag{2}$$

$f_h : \Re^n \times \Re^m \times N_{\geq 0} \rightarrow E \subseteq \{0,1\}^{n_e}$

- **finite state machines** - represent the discrete part of the hybrid systems, which can be stated as:

$$x_d(k+1) = f_d(x_d(k), u_d(k), \delta_e(k)) \tag{3}$$

$x_d(k+1) \in X_d \subseteq \{0,1\}^{n_d}$, $x_d(k)$ - discrete state vector, $u_d(k)$ - discrete input, $\delta_e(k)$ - output of event generator

- **mode selector** - based on the output of the event generator and finite state machine it defines the continuous part of the dynamical system in the frame of switched affine systems. Mode selector output signal is defined in the form:

$$p(k) = f_m(x_d(k), u_d(k), \delta_e(k)) \tag{4}$$

Further we will mainly deal with the discrete hybrid automaton represented by discrete PWA systems. Actual mode of these systems depends on actual position of the state vector in specific area only. A switched affine system can be rewritten as the combination of linear terms and if-then-else rules. The first equation in (1) is then equivalent to the form [8]:

$$z_1(k) = \begin{cases} A_1 x(k) + B_1 u(k) + f_1; & \text{if } (p(k) = 1) \\ 0; & \text{otherwise} \end{cases} \tag{5}$$

$$z_r(k) = \begin{cases} A_r x(k) + B_r u(k) + f_r; & \text{if } (p(k) = r) \\ 0; & \text{otherwise} \end{cases} \tag{6}$$

$$x(k+1) = \sum_{i=1}^{r} z_i(k) \tag{7}$$

where $z_i(k) \in \Re^n, i = 1, \ldots, r$. The same procedure is applied also on the second equation in (1).

III. Modelling of the Coupled Tanks Hydraulic Hybrid System

Consider a hybrid hydraulic dynamical system stated in [6], which structure is shown in Fig. 4. The input to the system represents the inflow $q_0(t)$. Dynamics of this system is dependent on the height of a liquid in the second tank $h_2(t)$. If the height $h_2(t)$ is lower than the height h, in which the liquid level in the second tank runs over the the height of the bottom of the first tank, the dynamics evolves according to the physical laws for the two tank hydraulic dynamical system without interaction (8)-(9). However if the height $h_2(t)$ is above the height h, the dynamics of the system is described as the two tanks hydraulic system with interaction (10)-(11).

Parameters of the mentioned hydraulic system are: $R_1\,[m]$ - radius of the sphere tank, $F_2\,[m^2]$ - cross-sectional area of the second tank, $k_1, k_2\,[m^{5/2}s^{-1}]$ - flow resistances constants

Mode $p = 1$ ($h_2(t) < h$, without interaction)

$$\dot{h}_1(t) = \frac{q_0(t)}{\pi(2R_1 h_1(t) - h_1^2(t))} - \frac{k_1\sqrt{h_1(t)}}{\pi(2R_1 h_1(t) - h_1^2(t))}, \tag{8}$$

$$\dot{h}_2(t) = \frac{k_1\sqrt{h_1(t)}}{F_2} - \frac{k_2\sqrt{h_2(t)}}{F_2} \tag{9}$$

Mode $p = 2$ ($h_2(t) >= h$, with interaction)

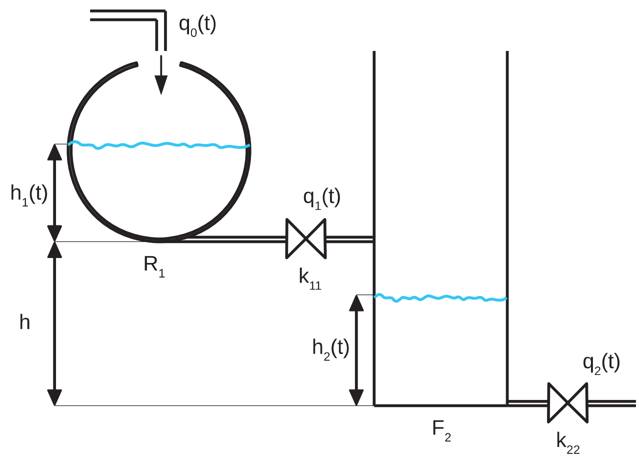

Fig. 4. The graphical description of coupled tanks hydraulic system

$$\dot{h}_1(t) = \frac{q_0(t)}{\pi(2R_1h_1(t)-h_1^2(t))} - \frac{sign(L(t))k_1\sqrt{|L(t)|}}{\pi(2R_1h_1(t)-h_1^2(t))}, \tag{10}$$

$$\dot{h}_2(t) = \frac{\text{sign}(L(t))k_1\sqrt{|L(t)|}}{\pi(2R_1h_1(t)-h_1^2(t))} - \frac{k_2\sqrt{h_2(t)}}{F_2} \tag{11}$$

where $L(t) = h_1(t) - (h_2(t) - h)$.

These two modes of the stated hybrid hydraulic system dynamics represent finite state machines. A set of nonlinear differential equations is assigned to each of these modes. Fulfilment of the condition $h_2(t) = h$ triggers event generator and consequently mode selector selects the dynamics of the system in the switched affine systems frame.

After obtaining a nonlinear hybrid system model we performed a linear approximation in the close vicinity of the steady states of the dynamical system for both dynamics modes in order to use linear control design. Subsequently, discretizing the dynamical system according to Euler method [12] with the sample rate T was realized (12), so that the dynamical system could be implemented within HYSDEL modelling framework.

$$h_{1,2}(k+1) = h_{1,2}(k) + Tf(h_{1,2}(k), q(k)) \tag{12}$$

Output of this procedure is the discrete PWA description of the hybrid dynamical system in (13)-(14) in the form (5)-(7). To shorten the difference equations we labeled cross sectional area of the first tank as $F_1(t) = \pi(2R_1h_1(t) - h_1^2(t))$ and its linearized representations F_{11}^s, F_{12}^s which denote cross-sectional areas of the first tank in its steady state for the specified discrete mode.

Mode $p = 1$ ($h_2(k) < h$, without interaction)

$$\begin{aligned}\Delta h_1(k+T) &= \left(1 - \frac{k_{11}T}{F_{11}^s}\right)\Delta h_1(k) + \frac{\Delta q_0(k)T}{F_{11}^s} \\ \Delta h_2(k+T) &= \frac{k_{11}T}{F_2}\Delta h_1(k) + \left(1 - \frac{k_{21}T}{F_2}\right)\Delta h_2(k)\end{aligned} \tag{13}$$

Mode $p = 2$ ($h_2(k) >= h$, with interaction)

$$\begin{aligned}\Delta h_1(k+T) &= \left(1 - \frac{k_{12}T}{F_{12}^s}\right)\Delta h_1(k) + \frac{k_{12}T}{F_{12}^s}\Delta h_2(k) + \\ &+ \frac{\Delta q_0(k)T}{F_{12}^s} \\ \Delta h_2(k+T) &= \frac{k_{12}}{\frac{F_1^s}{T}}\Delta h_1(k) - \left(\frac{k_{12}}{\frac{F_2}{T}} + \frac{k_{22}}{\frac{F_2}{T}} - 1\right)\Delta h_2(k)\end{aligned} \tag{14}$$

Parameters of this discrete PWA representation of the nonlinear hybrid system are: $k_{11}, k_{12}, k_{21}, k_{22}$ - modified flow resistances constants for each discrete mode and T - evaluated sample rate according to the dynamics of the system.

Based on the linear description of the hybrid hydraulic system (13)-(14) we can extract state-space matrices $F_i, G_i, C_i, i = 1, 2$ defined in (15) and (16).

$$F_1 = \begin{bmatrix} 1 - \frac{k_{11}T}{F_{11}^s} & 0 \\ \frac{k_{11}T}{F_2} & 1 - \frac{k_{21}T}{F_2} \end{bmatrix}, G_1 = \begin{bmatrix} \frac{T}{F_{11}^s} \\ 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{15}$$

$$F_2 = \begin{bmatrix} 1 - \frac{k_{12}T}{F_{12}^s} & \frac{k_{12}T}{F_{12}^s} \\ \frac{k_{12}T}{F_2} & 1 - \frac{k_{12}T}{F_2} - \frac{k_{22}T}{F_2} \end{bmatrix}, G_2 = \begin{bmatrix} \frac{T}{F_{12}^s} \\ 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{16}$$

These matrices will be used for optimal LQR linear synthesis in the following section.

IV. Optimal Control Based on LQR Synthesis for Dynamical Systems with Hybrid Dynamics

This section describes application of the LQR synthesis on the hybrid system in the discrete PWA representation. The state space of the system is partitioned according to switching dynamics conditions. For each partition in the state space one LQR and feed-forward controller is computed. LQR controller provides stabilization of the system into its equilibrium point in each discrete mode. The feed-forward controller serves to follow the reference trajectory regardless to the discrete mode of the system. The control scheme implementing these control objectives is depicted in Fig. 5.

The final control law $\Delta u(k)$ is composed of two parts: the feedback part $\Delta u_{fb}(k)$ and feed-forward part $\Delta u_{ff}(k)$, whose description was mentioned above:

$$\Delta u(k) = \Delta u_{fb}(k) + \Delta u_{ff}(k) = -\mathbf{k}_i\Delta\mathbf{x}(k) + N_i\Delta w(k) \tag{17}$$

where $\mathbf{k}_i$ is the state-feedback gain which minimizes the quadratic cost function $J_i(u)$ [13]:

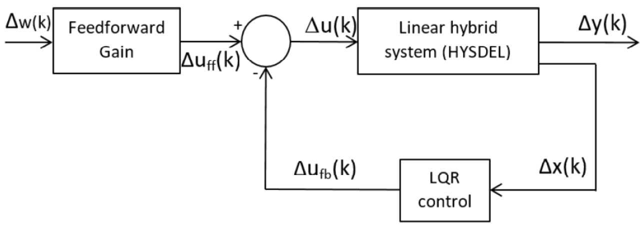

Fig. 5. Control scheme for discrete LQR control with feedforward gain

$$J_i(u) = \sum_{n=1}^{\infty} \left(\Delta\mathbf{x}^T(k)\mathbf{Q}_i\Delta\mathbf{x}(k) + \Delta u_{fb}^T(k)\mathbf{R}_i\Delta u_{fb}(k) \right) \tag{18}$$

for each discrete mode $p(k)$. N_i represents feed-forward gain, which is computed as:

$$N_i = \frac{1}{C_i(I-(F_i-G_ik_i))^{-1}G_i}, i = 1, 2 \tag{19}$$

The hybrid control law (17) is then applied on the nonlinear hybrid dynamical system and switching between controllers is performed according to the change of active partition in the state-space.

V. Implementation and Validation of the Hybrid Control Model within Simulation Tools

This section describes necessary steps and modelling frameworks which we used for the implementation of the nonlinear and linear representations of the hybrid hydraulic system as well as the validation of the proposed optimal control.

A. Comparison of the Nonlinear and Discrete PWA Hybrid System

In this part of the section we will verify discrete PWA form of the nonlinear hybrid system. It is useful to model and simulate the nonlinear hybrid hydraulic system shown in Fig. 4 by utilization of s-functions. S-functions allow to define differential equations into Simulink with the help of MATLAB environment m-files and are built with the help of Mex utility. This means that they represent dynamically interconnected sub-programs which can be automatically read and run by MATLAB. Utilizing s-functions we modelled the nonlinear hybrid hydraulic system represented by coupled tanks (8)-(11).

Modelling framework HYSDEL (HYbrid System DEscription Language) is a free software tool which can be used within MATLAB environment. Preferred application of HYSDEL is the area of modelling and control of DHA systems [8]. Syntax structure of HYSDEL consists of two main parts, specifically INTERFACE and IMPLEMENTATION [5]. The part INTERFACE serves as a declaration of variables and contains these subparts:

- STATE - declares state variables and their range
- INPUT - declares input variables and their range
- PARAMETER - declares system parameters and their range
- OUTPUT- declares output variables and their range

and the part IMPLEMENTATION defines relations between declared variables [8] and consists of these subparts:

- AUX - declaration of auxiliary variables, needed for calculations in the IMPLEMENTATION section
- AD - analog-digital block, specifying relations between variables of type REAL to BOOL
- DA - digital-analog block, specifying relations between variables of type BOOL to REAL
- LOGIC - logical relations between variables of type BOOL
- LINEAR - linear relations between variables of type REAL
- CONTINUOUS- state update equation for variables of type REAL
- AUTOMATA - state update equation for variables of type BOOL
- OUTPUT - selection of output variables which can be of type REAL or BOOL
- MUST - specification of input/state/output constraints

We used HYSDEL to model discrete PWA representation (13)-(14) of the nonlinear hybrid hydraulic system. The first HYSDEL part INTERFACE specifies all the system inputs, outputs, states and parameters as well as their ranges.

As far as the second HYSDEL part IMPLEMENTATION is concerned, we do not use all of its subparts. Subpart AUX contains all auxiliary variables needed in the next subparts. Dynamics switching condition $h_2(t) < h$ is implemented in AD subpart. Subpart DA implements discrete PWA representation of the hybrid hydraulic system in the form (5)-(6). Subpart CONTINUOUS specifies final system dynamics according to (7). Finally subpart OUTPUT implements output matrices $C_i, i = 1, 2$ in (15) and (16). Subpart MUST does not need to be specified since all the ranges of the states, imputs, etc. were declared in the part INTERFACE.

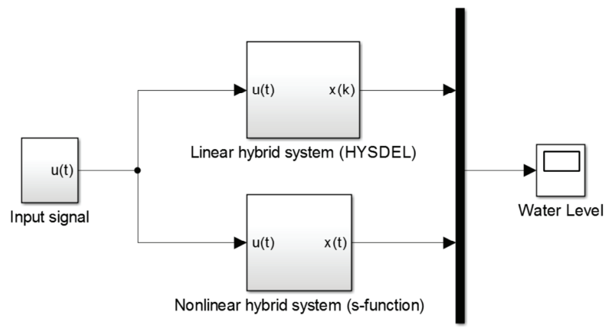

Fig. 6. Scheme for validation of the discrete PWA representation of the nonlinear hybrid system

After implementation of both hybrid hydraulic system descriptions we verified the discrete PWA representation within the simulation scheme in Fig. 6. It is important to notice that the scheme does not contain deviation model of the linear system. The steady water levels and inflow are implemented within HYSDEL modelling framework and therefore they must not be also implemented within Simulink environment. Another thing to notice on this validation scheme is the continuous input *u(t)* and discrete output *x(k)* for the HYSDEL representation of the hybrid system. As long as the HYSDEL structure requires the parameter sample rate *T*, it samples the input signal *u(t)* according to this sample rate into the form *u(k)* within the HYSDEL block.

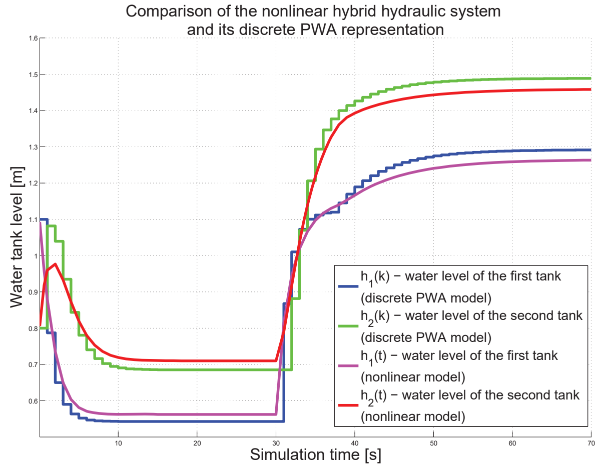

Fig. 7. Validation of the nonlinear and discrete linear representation of the hybrid sytem

In Fig. 7 we can see that the discrete linear approximation of the original system copies the behaviour of the nonlinear system really well and can be used for the linear optimal control synthesis.

B. LQR Synthesis for the Hydraulic Discrete PWA System

This subsection deals with LQR synthesis based on the discrete PWA description and subsequent control law validation on the nonlinear hybrid system. Based on the state-space partitioning in Fig. 8, and weight matrices $\mathbf{Q}_i$, $\mathbf{R}_i$ for each discrete mode, we computed feedback and feed-forward gains for each discrete mode of the discrete PWA representation.

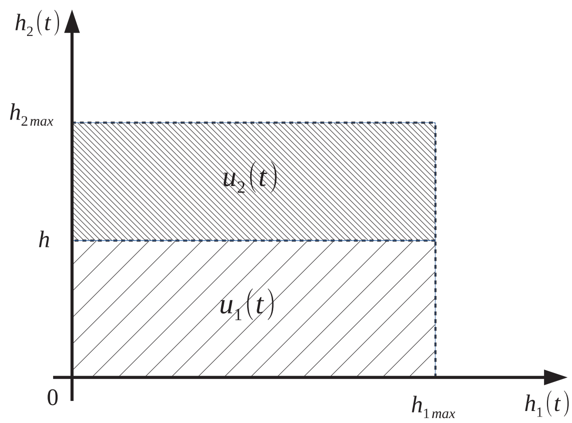

Fig. 8. State-space partitioning of the hybrid system represented by hybraulic coupled tanks

Then the optimal control laws $u_1(t)$ and $u_2(t)$ were tested on the implemented hybrid linear system within HYSDEL modelling framework and subsequently on the hybrid nonlinear system. The structure of the simulation experiment is shown in Fig. 9.

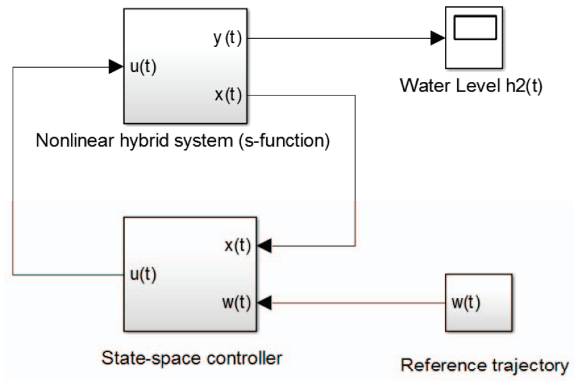

Fig. 9. Control scheme of simulation experiment for optimal control of hybrid system

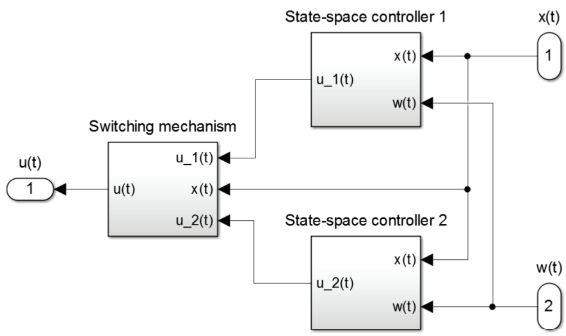

Fig. 10. Scheme of switching state-space controller based on LQR synthesis

To illustrate the implementation of the *State-space controller* block within Simulink environment in Fig. 9, its decomposition and implementation is shown in Fig. 10.

After application of the proposed hybrid optimal control on the hybrid dynamical system, which objective was to follow the reference trajectory we obtained results depicted in Fig. 11. Switching water level height was set to $h = 1.35m$.

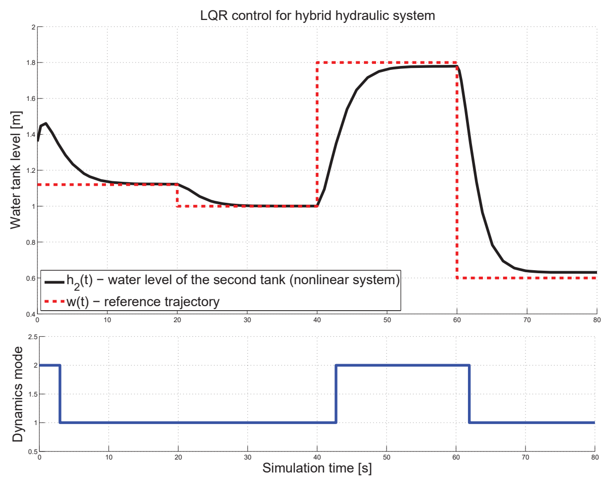

Fig. 11. Application of the linear LQR synthesis on the nonlinear hybrid system represented by hydraulic coupled tanks

From the time behaviour of the second tank water level $h_2(t)$ it is obvious that the initial conditions were not set into the first setpoint of the reference trajectory. After some time the water level reached reference trajectory. In the second half of the simulation experiment results (simulation time 40s - 80s) it can be seen that the reference trajectory is further from the hydraulic system steady-points, that causes permanent error between the reference trajectory $w(t)$ and water tank level $h_2(t)$.

VI. Conclusion

In this paper we introduced one of the CPS challenges, which is hybrid modelling of such systems. We provided a case study of a nonlinear hybrid coupled tanks system. The system captures both the continuous and the discrete dynamics. Subsequently we reformulated the mathematical model of the nonlinear system into the discrete PWA representation. This form was chosen because of using HYSDEL modelling framework for the hybrid systems. Utilizing this modelling framework was convenient in terms of automatic switching of the discrete dynamics modes of the system.

After successful validation of the linear representation of the nonlinear hybrid system we designed control strategy based on LQR and feed-forward gain synthesis. Two final control laws were provided, each for one of the discrete dynamics modes. Switching between the control laws was ensured by checking the state-space vector of the system.

Subsequently after applying these prepositions the optimal hybrid control law was first tested on the linear representation of the system and then also on the former nonlinear representation of the system.

The next challenges of our research group (*Center of Modern Control Techniques and Industrial Informatics*) focus on modelling more complex hybrid systems, for example by introducing discrete(boolean) inputs, outputs and/or states. All of these extensions can be applied on the hybrid hydraulic system presented in this paper and will be further studied. Another challenge lies in controlling the hybrid system, e.g. explicit model predictive control and comparison of such an approach against proposed optimal control based on LQR synthesis.

Acknowledgment

This work has been supported by the Research and Development Operational Program for project: Innovation Applications Supported by Knowledge Technology, ITMS: 26220220182, co-financed by the ERDF (70%) and project KEGA - 001TUKE-4/2015 (30%).

References

[1] A. Bemporad and M. Morari. Control of systems integrating logic, dynamics, and constraints. *Automatica*, 35(3):407–427, 1999.

[2] P. Collins and J. H. Van Schuppen. Observability of piecewise-affine hybrid systems. In *Hybrid Systems: Computation and Control*, pages 265–279. Springer, 2004.

[3] M. Fikar and T. Hirmajer. Dynamic optimization of a hybrid coupled tanks system. *Journal of Electrical Engineering*, 57(3):167–172, 2006.

[4] C. S. Group et al. Cyber-physical systems executive summary. *CPS Summit*, 2008.

[5] M. Herceg. *Real-Time Explicit Model Predictive Control of Processes*. PhD thesis, ÚIAM FCHPT STU v Bratislave, Radlinského 9, 812 37 Bratislava, 10.12.2009 2009.

[6] V. Hladký, L. Popovic, and J. Sarnovský. Modeling of a system with hybrid dynamics. *Acta Electrotechnica et Informatica*, 11(1):14–19, 2011.

[7] A. Jadlovská, S. Jadlovská, and D. Voščček. Cyber-physical system implementation into the distributed control system (preprint). In *14th IFAC International conference on Programmable devices and embedded systems*, pages 375–380, 2016. ISSN : 2405-8963.

[8] M. Kvasnica. *Efficient software tools for control and analysis of hybrid systems*. PhD thesis, Diss., Eidgenössische Technische Hochschule ETH Zürich, Nr. 17662, 2008.

[9] E. A. Lee and S. A. Seshia. *Introduction to Embedded Systems*. Citeseer, 2014.

[10] D. Liberzon. *Switching in systems and control*. Springer Science & Business Media, 2012.

[11] J. Malmborg. *Analysis and design of hybrid control systems*. PhD thesis, 1998.

[12] A. Quarteroni, R. Sacco, and F. Saleri. *Numerical mathematics*, volume 37. Springer Science & Business Media, 2010.

[13] J. Sarnovský, A. Jadlovská, and P. Kica. *Optimal and Adaptive Systems Theory*. Košice: ELFA s.r.o., 2005.

[14] D. Voščček. Cyber-physical systems, modelling framework and application into distributed control systems. In *SCYR 2016*, pages 88–91. TU, 2016.

[15] J. Wan, H. Yan, H. Suo, and F. Li. Advances in cyber-physical systems research. *TIIS*, 5(11):1891–1908, 2011.

[16] Q. Wang, G. Yang, X. Zhou, and Y. Yang. Behavior modeling of cyber-physical system based on discrete hybrid automata. In *Computational Science and Engineering (CSE), 2013 IEEE 16th International Conference on*, pages 680–684. IEEE, 2013.

[17] Y. Yang and X. Zhou. Cyber-physical systems modeling based on extended hybrid automata. In *Computational and Information Sciences (ICCIS), 2013 Fifth International Conference on*, pages 1871–1874. IEEE, 2013.