Design and Simulation of Control for Nonlinear Model Ball&Beam

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***Abstract* — This paper deals design of control algorithms using approximate linearization method, namely PD and state control algorithm for nonlinear simulation model Ball&Beam. In this paper is given a mathematical – physical model whereby was created nonlinear simulation model Ball&Beam in the Simulink environment. The proposed control algorithms are implemented into control structures with the aim tracking desired position of ball along beam. The programmed simulation schemes of the control structures for tracking reference trajectory are verified in the Matlab/Simulink language.**

***Keywords* — nonlinear model, pole – placement method, PD control algorithm, state control algorithm.**

I. Introduction

Model Ball&Beam is the typical example of a natural nonstable system, which is mostly used in teaching, based on its properties, in the validation and testing of various proposed control algorithms in control structures [2].

In this paper are designed PD and state control algorithm for control of the simulation nonlinear model, which is based on mathematical – physical model Ball&Beam. The proposed control algorithms are implemented into simulation scheme in Maltab/Simulink language with the aim to track desired trajectory, which represents the position of ball along beam.

The control algorithms based on linear model in input – output or state description were used for comparison with algorithms, which are designed by nonlinear method of synthesis of exact linearization in my work to the dissertation exam for model Ball&Beam [8].

II. Mathematical – physical model Ball&beam

The model Ball&Beam (B&B) (Fig.1) is nonlinear dynamic SISO model with one input (u_φ – voltage that control servomotor) and one output (r – position of ball along beam).

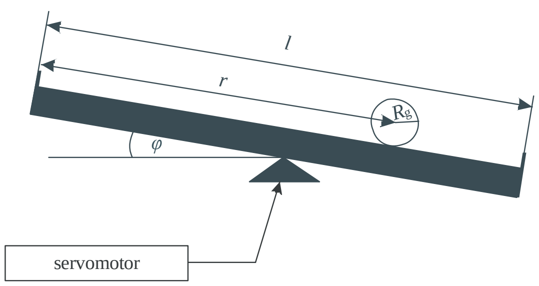

Fig. 1 Ball&Beam model

Model B&B is a simplification of Ball&Plate (B&P) model, which is located at the Laboratory of Cybernetics at the Department of Cybernetics and Artificial Intelligence [1].

The model B&B is divided into two subsystems namely subsystem *Movement of ball along beam* and subsystem *Servomotor* (Fig.2). To detect of position of ball along beam is added sensor to the model.

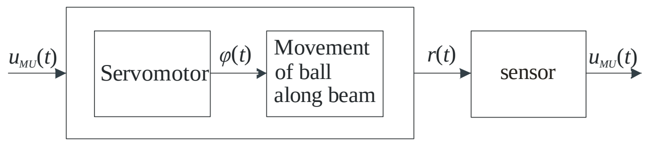

Fig. 2 Ball&Beam – subsystems

In compiling of the mathematical – physical model of subsystem *Movement of ball along beam* is based on the Euler - Lagrange equations in the shape

$$\frac{d}{dt}\left\{\frac{\partial}{\partial \dot{q}_i(t)} L(t)\right\} - \frac{\partial}{\partial q_i(t)}\{L(t)\} = \frac{\partial}{\partial q_i}\{W(t)\} \tag{1}$$

where $L(t) \equiv L = E_k - E_p$ is Lagrangian, $q_i(t)$ is i – th generalized parameter and $W(t)$ is work, that undertaken by the nonconservative forces. The result after determination of total kinematic E_k and potential energy E_p and defining generalized parameters ($q_1(t) = r(t)$ *a* $q_2(t) = \varphi(t)$) are two nonlinear differential equations

$$\left[m_g + \frac{J_g}{R_g^2}\right] \cdot \ddot{r}(t) - m_g \cdot r(t) \cdot \dot{\varphi}^2(t) + m_g \cdot g \cdot \sin(\varphi) = 0 \tag{2}$$

$$\left[J_t + m_g \cdot r^2(t)\right] \cdot \ddot{\varphi}(t) + m_g \cdot g \cdot r(t) \cdot \cos(\varphi) = F_t \cdot l \cdot \cos(\varphi) \tag{3}$$

where $r(t)$ is current position of the ball, $\varphi(t)$ is angle of the rotation of the beam, F_t is force acting in direction of the beam, m_g is mass of the ball, R_g is radius of the ball, J_g is moment of inertia of the ball, J_t is moment of inertia of the beam and g is acceleration of gravity. The equation (2) expresses impact angle of the beam on the movement of the ball and equation (3) expresses impact ball on the angle of the beam, which neglects. Programmed simulation scheme of the subsystem *Movement of ball along beam* is in Fig.3.

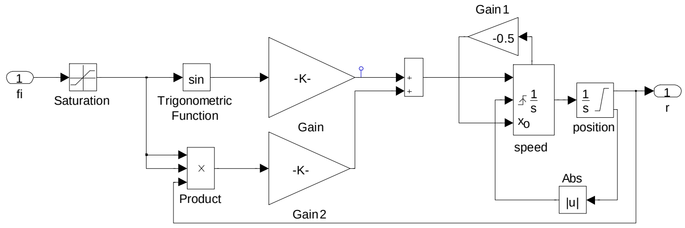

Fig. 3 Simulation scheme of the subsystem of Movement of ball along beam

A complete description of the model B&B is still necessary to define subsystem *Servomotor*, which can be described nonlinear differential equation in the shape

$$\frac{d\varphi(t)}{dt} = \omega.sign[u_{MU}(t) - u_{\varphi}(t)] \text{ pre } [u_{MU}(t) - u_{\varphi}(t)] \geq sens \quad (4)$$

resp.

$$\frac{d\varphi(t)}{dt} = 0 \text{ pre } [u_{MU}(t) - u_{\varphi}(t)] < sens \quad (5)$$

where ω is nominal angular velocity of the servomotor and *sens* is characteristic for nonlinear element type three – position relay. Simulation scheme of the subsystem *Servomotor* (Fig.4) was programmed from nonlinear differential equations (4) and (5). [1], [2]

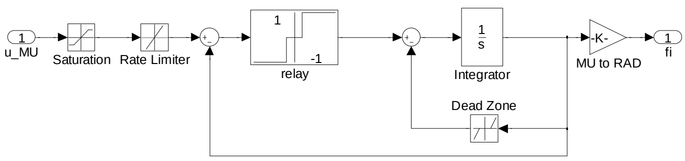

Fig. 4 Simulation scheme of the subsystem of Servomotor

III. DESING PD CONTROL ALGORITHM

For proposal control algorithms using approximate linearization method is necessary linearized nonlinear model B&B in point [0,0]. In this case is necessary linearized both subsystems *Movement of ball along beam* (2) and *Servomotor* (4), (5).

After treatment will have nonlinear differential equation (2) shape

$$\ddot{r}(t) = K_m \sin\varphi(t) \quad (6)$$

Whereas the rolling of the beam is limited in range ± 5% for linearization of equation (6) is used fact that sin $\varphi \cong \varphi$ for the small angles, then transfer function of the subsystem *Movement of ball along beam* is in the shape

$$\ddot{r}(t) = K_m \varphi(t) \quad \rightarrow \quad F_m = \frac{Y(s)}{U(s)} = \frac{K_m}{s^2} \quad (7)$$

Subsystem *Servomotor* (Fig.4) contains hard nonlinearities and therefore can not linearized equation (4) and (5) using Taylor series. Therefore method of gradual integration was used to obtain a linear approximation of the subsystem *Servomotor* [4].

After applying algorithm of the identification method was obtained linear approximation of the subsystem Servomotor in the shape

$$F_s = \frac{K_s}{T_s s + 1} \quad (8)$$

The sensor with static characteristic, where the sensor's constant is K_{sn}, is used to detect position of the ball along beam. The resulting transfer function of the model B&B is then in the shape

$$F = F_{sn} F_m F_s = \frac{K_{sn} K_m K_s}{s^2 (T_s s + 1)} = \frac{K_{B\&B}}{s^2 (T_s s + 1)} \quad (9)$$

The obtained linearized model of input – output description (9) is used to design the PD control algorithm with aim to track the given trajectory, that includes control on the equilibrium [0,0] and also control on the steady state.

The aim is design the control algorithm, which ensures that control error converge to zero during tracking reference trajectory i.e. $e(t) \rightarrow 0$.

Based on the resulting transfer function of the model B&B (9) was proposed the parameters for PD control algorithm using programmed algorithm of method of standard shapes of Graham – Lantrop for consider control law in the form

$$u(t) = K(e(t) + T_d \frac{de(t)}{dt}) \quad (10)$$

The proposed simulation scheme of closed – loop system is in Fig.5, where is implemented the proposed PD control algorithm to control nonlinear model B&B.

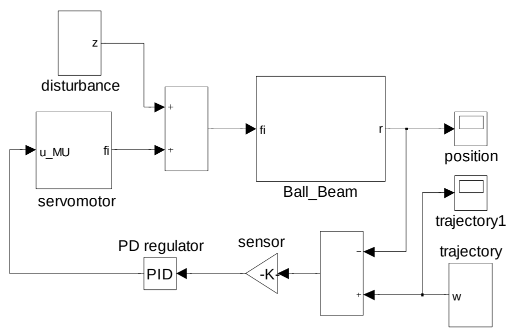

Fig. 5 Simulation scheme of feedback control structure using the PD control for the model B&B

The resulting graph of the tracking reference trajectory using PD control algorithm is in Fig. 6, from which is seen that output of the nonlinear model B&B tracks reference trajectory. The oscillations around steady states are caused to the fact that the proposed control is applied to the nonlinear model.

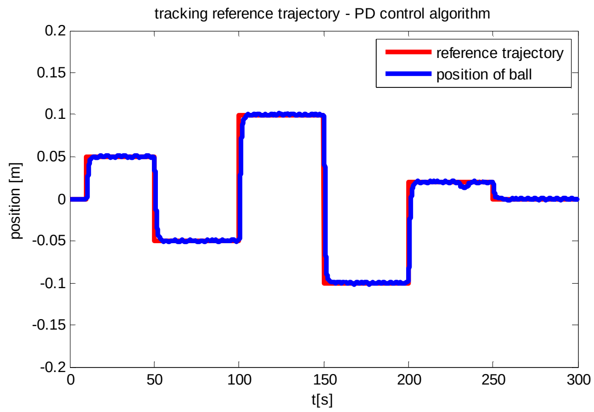

Fig. 6 Tracking reference trajectory – PD control algorithm

IV. DESING STATE CONTROL ALGORITHM

The another proposal control algorithm using approximate linearization method is state control algorithm, which requires a linear model B&B (9) in state space

$$\begin{aligned} x(t) &= Ax(t) + bu(t) \\ y(t) &= c^T x(t) \end{aligned} \tag{11}$$

where state matrix of the model B&B are consider in the shape

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & K_{B\&B} \\ 0 & 0 & -\frac{1}{T_s} \end{bmatrix},\ b = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_s} \end{bmatrix},\ c^T = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{12}$$

Base on the matrix A, b, c^T of the state description of the model B&B, suitable chosen roots $p = [p_1, p_2, p_3]$ and using function of Control Toolbox *place* based on pole – placement method is designed vector of gains K for consider control law in the form

$$u(t) = -Kx(t) + Nw(t) \tag{13}$$

The gain N of the control algorithm, which ensures tracking trajectory is determined from the relation [7]

$$N = \frac{1}{c^T (I - (A - bK))^{-1} b} \tag{14}$$

The proposed control algorithm ensures control on the equilibrium but it not sufficient for control on the steady state, therefore the chosen roots p were optimized using programmed genetic algorithm yet, that was achieved the desired behavior of the nonlinear simulation model B&B. [9]

The proposed simulation scheme using the designed state control for nonlinear model B&B in the feedforward control structure is in Fig.7.

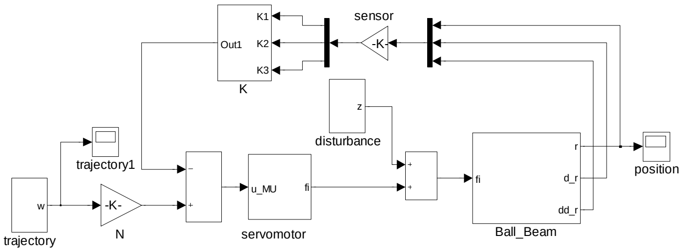

Fig. 7 Simulation scheme of control structure with forward regulator using state control algorithm

The resulting graph of tracking reference trajectory of model B&B using state control is in Fig.8. The reference trajectory is the same as when was used PD control algorithm. The Fig.8 shows that output of the model B&B tracks the desired trajectory but when compared with graph of using PD control (Fig.6) are visible the larger oscillation around the steady state.

The proposed PD and state control algorithm are verified on the nonlinear simulation model B&B, which created conditions for their further use for real model Ball&Plate. The model B&P is divided into two subsystems namely subsystem for a axis x and subsystem for a axis y, while model B&B represent one of the subsystems its simpler variant of the model B&P. [1]

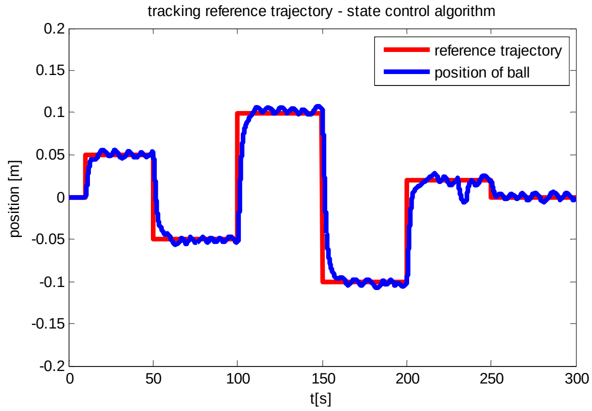

Fig. 8 Tracking reference trajectory – state control algorithm

V. CONCLUSION

The paper presents the design of PD and state control algorithm for nonlinear simulation model B&B. The approximate linearization of the nonlinear model is used in the both proposals and based on the obtained linear approximation of system is designed control for nonlinear model. These proposed control algorithms will serve as standard for comparison with the proposed algorithms using nonlinear methods of synthesis, namely exact linearization, gain – scheduling method and control based on the Ljapunov function, for model B&B resp. B&P in my future dissertation work, where the main aim is create a software tool for design of control algorithms for nonlinear physical systems.

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