Modeling and Control Design of Magnetic Levitation System

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Nonlinear Model of the Magnetic Levitation System

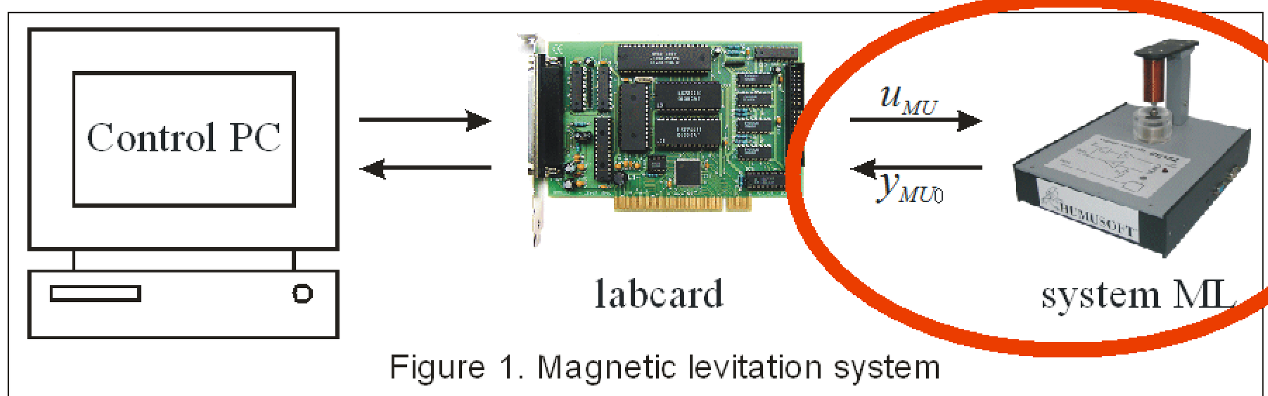

Figure 1. Magnetic levitation system

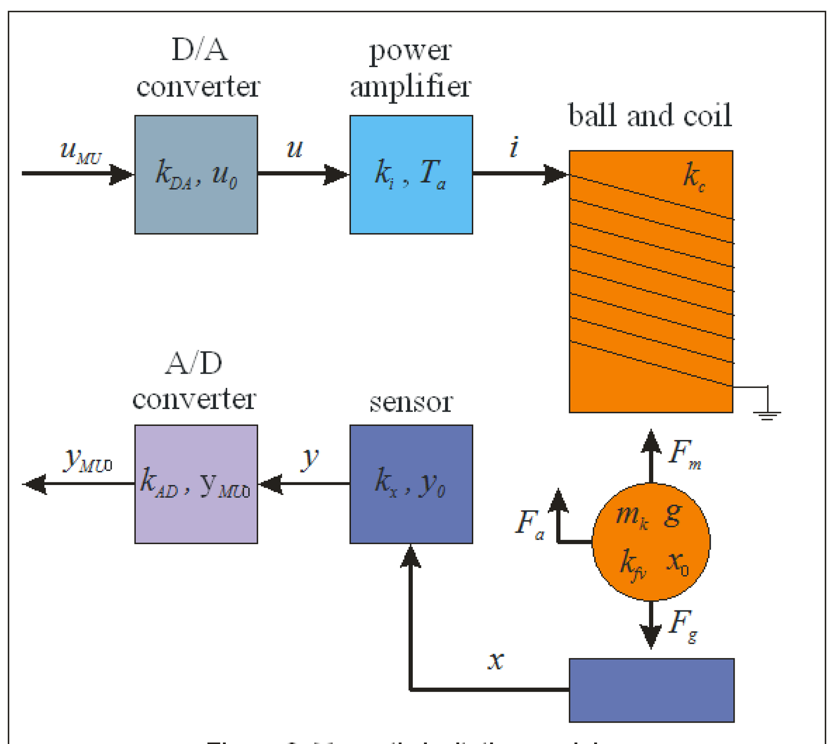

Figure 2. Magnetic levitation model

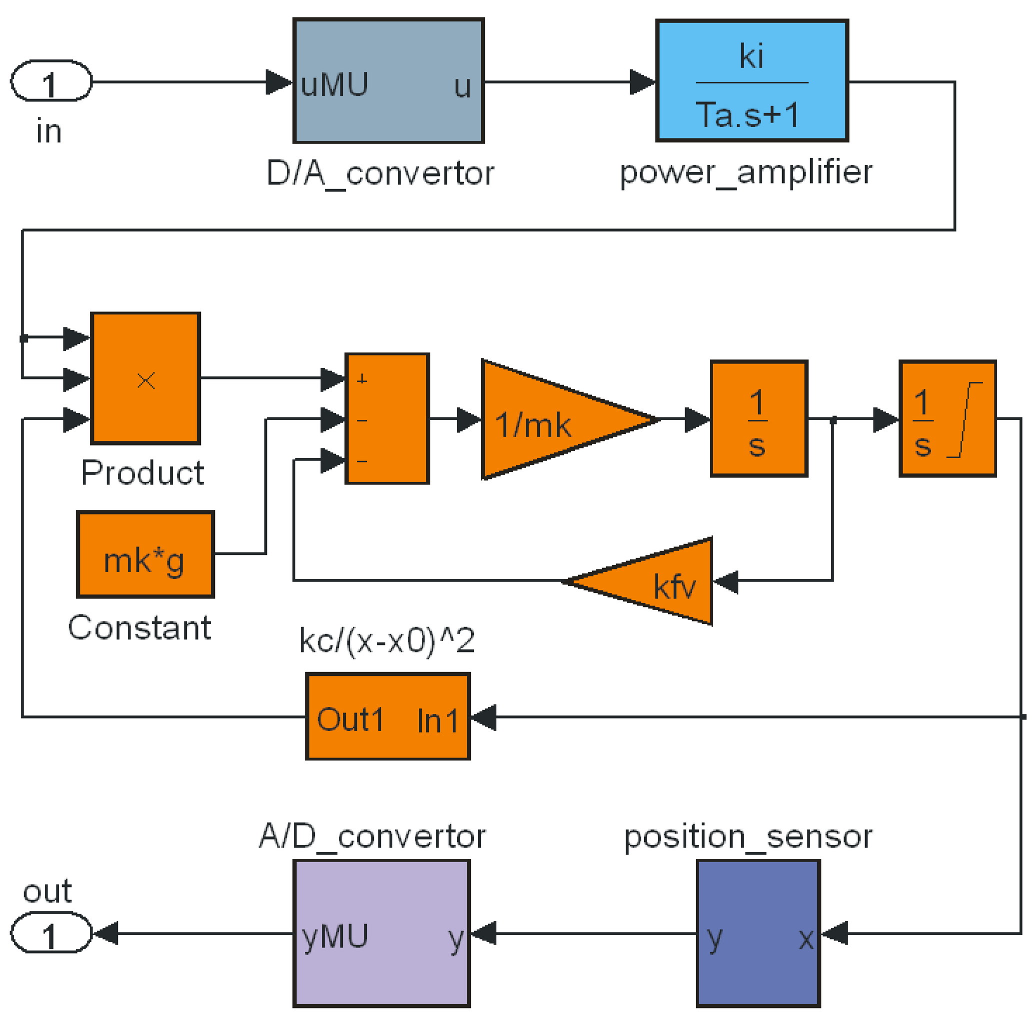

Figure 3. Simulation scheme of Magnetic levitation model

- i(t) - electric current [A]
- x(t) - ball position [m]
- y(t) - sensor output voltage [V]
- u(t) - converter output voltage [V]
- yMU(t)- converter output voltage [MU]
- y(t) - converter input voltage [V]
- uMU(t)- converter input voltage [MU
- mk - mass of ball [kg]
- kc - coil constant [A/V]
- x0 - coil offset [m]
- g - gravity constant [m/s-2]
- kfv - damping constant [N/m.s]
- ki - coil and amplified gain [A/V]
- Ta - coil and amplified time constant [s]
- kx - sensor gain [V/m]
- y0 - sensor offset [V]
- kDA - converter gain [V/MU]
- u0 - converter offset [V]
- kAD - converter gain [MU/V]
- yMU0- converter offset [MU]

Exact Input-Output Feedback Linearization Method

1. system written in affine representation

$$\dot{x}(t) = f(x,t) + g(x,t) * u(t)$$
$$y(t) = h(x,t)$$

2. first derivation $y(t)$

$$\dot{y} = \frac{\partial h}{\partial x} f + \frac{\partial h}{\partial x} gu = L_f h(x) + L_g h(x) u$$

3. if $L_g h(x) u \neq 0$ then $y^{r} = v$ and input transformation is

$$u = \frac{1}{L_g h(x)}(-L_f h(x) + v)$$

4. if $L_g h(x) u = 0$ then continues derivate of $y(t)$

$$\ddot{y} = L_f^2 h(x) + L_g L_f h(x) u$$

until $L_g L_f^{r-1} h(x) u \neq 0$ then $y^{r} = v$ and input transformation is

$$u = \frac{1}{L_g L_f^{r-1} h(x)}(-L_f^{r} h(x) + v)$$

5. to determine state transformation $z = T(x)$

$$z = T(x) = [h(x); L_f h(x), ..., L_f^{r-1} h(x)]$$

6. transformation of the system into linear form

$$\dot{z} = \begin{bmatrix} 0 & 1 & \dots & 0 \\ 0 & 0 & 1 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ \dots \\ 1 \end{bmatrix} v$$

7. for linear system is necessary to propose feedback control law by linear method of synthesis, to ensure the desired behavior of the system in case of change of the reference trajectory or for compensation disturbance.

8. input transformation (step . 4), state transformation (step . 5) and a control law (step 7) are implemented into control structure (Figure 4.)

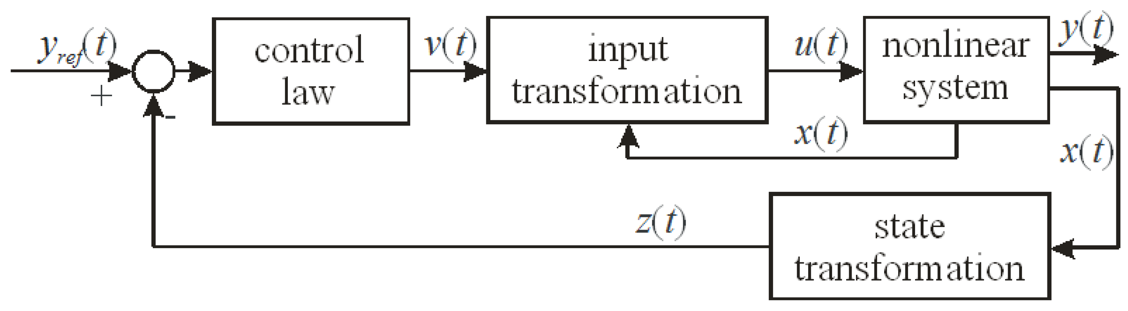

Figure 4. Control structure using exact feedback linearization method

Design of Nonlinear Control Algorithm

step 1.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ A\dfrac{x_3^2}{(x_1 - x_0)^2} - g + Bx_2 \\ -\dfrac{x_3}{T_a} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ C \end{bmatrix} u$$

$$y = Dx_1$$

$x = (x_1, x_2, x_3) = (x, \dot{x}, i)$, $u = u_{MU}$, $y = y_{MU}$, $A = \frac{k_c}{m_k}$, $B = \frac{k_{fv}}{m_k}$, $C = \frac{k_i k_{da}}{T_a}$, $D = k_{ad} k_x$

step 2-3.

$y = f(x_1),\ \dot{y} = f(x_2), \ddot{y} = f(x_1, x_2, x_3),$

$$\ddot{y} = \underbrace{\frac{2ADx_2x_3^2}{(x_0 - x_1)^3} + BD\left(bx_2 - g + \frac{ax_3^2}{(x_0 - x_1)^2}\right) - \frac{2ADx_3^2}{T_a(x_0 - x_1)^2}}_{\alpha} + \underbrace{\frac{2ADx_3C}{(x_0 - x_1)^2}}_{\beta} u$$

step 4.

$$u = \frac{1}{\beta}(-\alpha + v)$$

step 5.

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix}$$

step 6.

$$\dot{z} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

step 7.

$$v = -Kz + Ny_{ref}$$

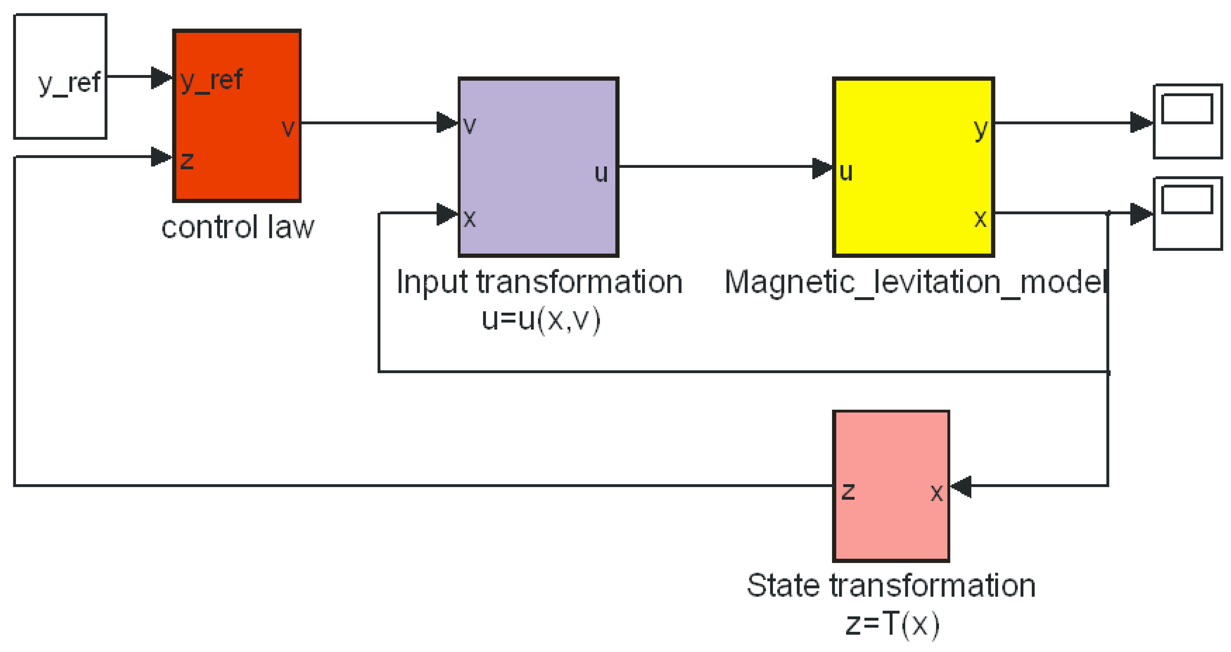

Figure 5. Simulation scheme for control of the nonlinear model of the Magnetic levitation using exact input output feedback linearization method

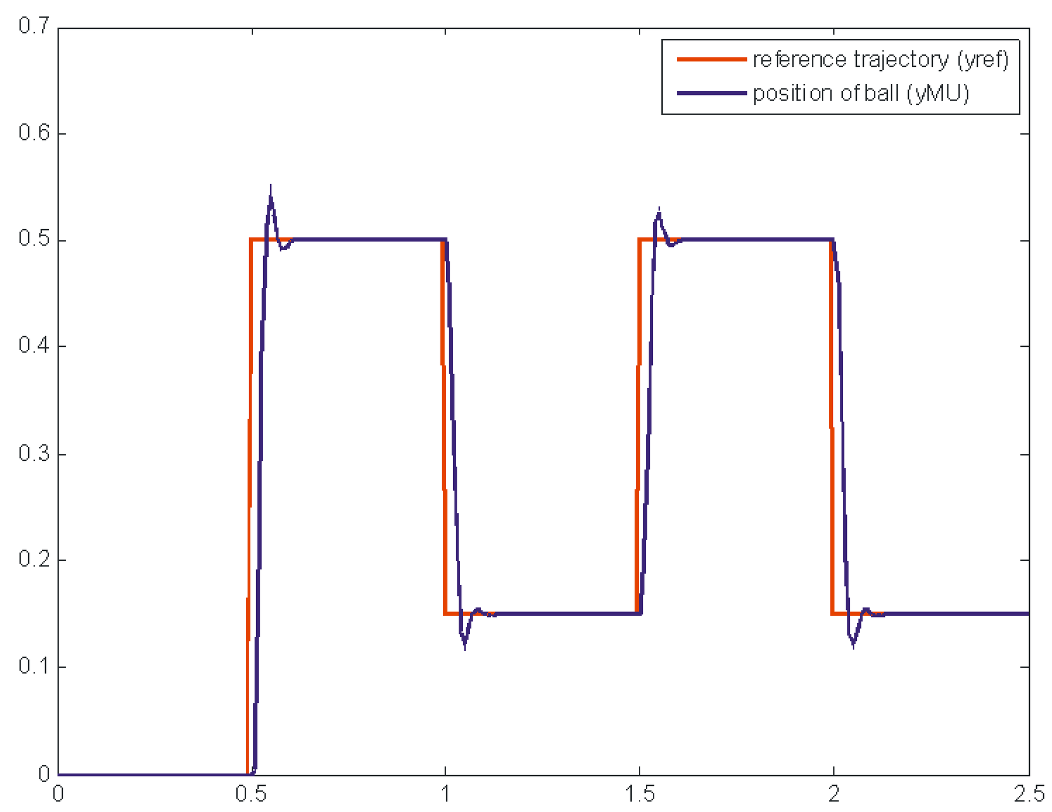

Figure 6. Magnetic levitation system response

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