Multi-objective optimization of modern assembly lines

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***Abstract*— This paper deals with various ways of using multi-objective (vector) optimization in assembly line optimization problem. Nowadays, there are two main approaches to solve multi-objective optimization tasks: artificial intelligence approach using mainly evolutionary algorithms; and mathematical approach based on detail mathematic description of optimized assembly line. This paper is summarizing knowledge of application possibilities of both approaches in assembly line optimizing. Also actual status of my dissertation thesis is mentioned, as well as list of solved and unsolved problems and future direction of my dissertation thesis.**

***Keywords*—assembly line, multi-objective optimization, objective function, constraints**

I. Introduction

Assembly line is a flow oriented production system consisting of productive units (workstations), which are aligned in a serial manner. The workpieces are transferred from one workstation to another via some kind of mechanical transport system (usually conveyor belt is used for this purpose).

Assembly lines were originally developed for cost efficient mass production of standardized products. First modern assembly line was designed by Henry Ford at the beginning of 20th century. This line was able to build Ford Model T in 93 minutes. However, since that time customer needs, as well as product requirements has changed dramatically. For example, German car producer, BMW, provides users with many additional features, which theoretically results in 10^{32} different models.

According to company requirements, designer is obligated to create assembly line, which is fulfilling customer's needs most. Assembly lines can be divided into many different groups considering various criteria, for example shape of assembly line. Some basic configurations of assembly lines can be found in Fig. 1.[1],[2]

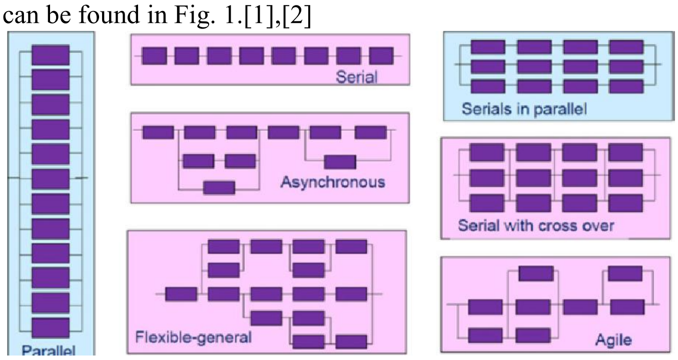

Fig. 1. Basic assembly line configurations

Assembly lines can produce only one simple model, or they can be adapted to produce number of different models. According to this criterion, assembly lines can be divided into simple, mixed and multi model assembly lines. Difference between them is displayed on Fig. 2.[3]

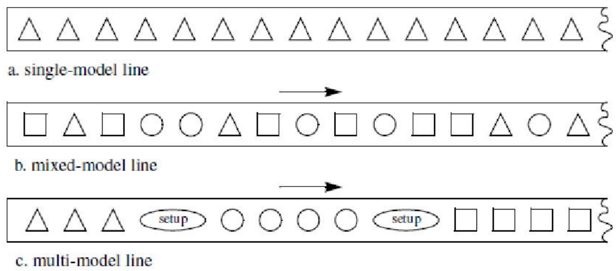

Fig. 2. Difference between single-model, mixed-model and multi-model assembly line

Some further information about mixed model assembly lines can be seen at [4] .Assembly lines and their workstations can be also divided into automatic, cooperative and manual. Other information about this topic and also about cooperation between humans and machines in assembly lines can be found in [5].

For any shape and any type of assembly line, there is always a need to use this line as efficiently as possible. Mentioned criteria, as well as many others, are influencing the way of access for optimization and balancing particular assembly line.

Assembly line balancing and optimization problem is represented by various optimization models aimed at supporting the decision maker in configuring efficient assembly system. Some problems and basic methods of assembly line balancing can be found in [6] and [3]. Ways of dealing with this problem are discussed in following chapters of this paper.

II. Multi-objective optimization

Optimization can be considered as choosing the best option from wide spectrum of alternatives. In our everyday life we are trying to make our work done spending as little time or as little energy as possible. Optimization methods are used to solve this problem most frequently, mostly because of their mathematical basis, which guarantees objectivity and accuracy of optimization process. Publications dealing with optimization problems and its solving are [7] and [8].

General optimization problem can be defined as minimization (maximization) of objective function

$$f = \{x_1, x_2, \dots, x_n\} \tag{1}$$

with respecting all constrains

$$g_i = \{x_1, x_2, \dots, x_n\}, for\ i = 1,2, \dots, n$$
$$x_j > 0, \qquad for\ j = 1,2, \dots, n. \tag{2}$$

For solving optimization problems, methods of mathematical programming are used. Depending on the type of objective function, these methods can be divided into:

- linear programming methods,
- non linear programming methods,
- integer programming methods,
- parameter programming methods,
- stochastic programming methods.

Multi-objective (vector) optimization is dealing with ways of optimizing the problem with multiple goals. It is used when it is necessary to accept more than one factor in finding the ideal solution of optimization task. This type of optimization was created for solving planning and organizing problems in manufacturing process. Nowadays it is used in many different areas (f.e.in dynamic management systems).

Vector optimization task is defined with relation to controlled system, which is described with n- dimensional vector $\boldsymbol{x} = (x_1, x_2, \dots, x_n x_1, x_2, \dots, x_n)$ with $x \in \{X\}$ representing n independent variables (decision variables). This system $\boldsymbol{x} \in \{X\}$is evaluated with k- dimensional vector functional

$$\boldsymbol{J}(\boldsymbol{x}) = (J_1(\boldsymbol{x}), J_2(\boldsymbol{x}), \dots, J_k(\boldsymbol{x})), \tag{3}$$

which elements are functions of vector $\boldsymbol{x}$, where k represents number of objective functions. This functional is optimized in subject to

$$g_i(x) \leq 0, i = 1,2, \dots, m, \tag{4}$$

and

$$h_l(x) = 0, l = 1,2, \dots, e, \tag{5}$$

where m is number of inequality constraints and e is number of equality constraints. Solution of this task is $\boldsymbol{x}^* \in \{X\}$, which is optimal solution of functional (or also called objectives, criteria, payoff functions, cost functions or value functions) $J_1(\boldsymbol{x}), J_2(\boldsymbol{x}), \dots, J_k(\boldsymbol{x})$ according to chosen type of vector optimization. [9],[10].

Basically, there are two main approaches to solve vector optimization tasks:

- artificial intelligence approach,
- mathematical approach.

A. *Artificial intelligence approach*

Main feature of artificial intelligence used in multi-objective optimization are evolutionary algorithms. They are used mainly for generating so called Pareto optimal set of solutions (solutions that are not dominated by any other solution of particular problem). Detail definition of Pareto optimality can be found in [9].

Genetic algorithms are part of evolutionary algorithms, which are characterized by a population of solution candidates. Reproduction process enables to combine existing solutions and generating new possible solutions. Finally there is a natural solution which determines which individuals of current population will participate in the next one. Functional description of evolutionary algorithms can be seen in Fig. 3.[11]

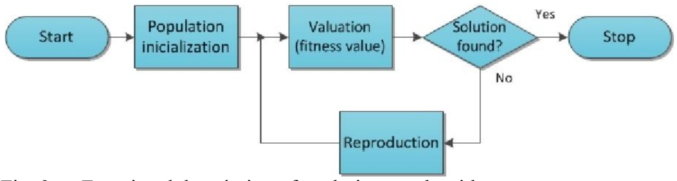

Fig. 3. Functional description of evolutionary algorithms

Some of evolutionary algorithms used for generating Pareto optimal set of solutions, but due to their working procedures, they often tend to stuck in good approximation and they do not guarantee identifying optimal trade-offs.

Artificial intelligence approach has wide application potential. For example, commonly used algorithm is ant colony algorithm. Its application in modeling and balancing time and space constrained assembly line can be found in [12]. Another application of ant colony algorithm is described in [13], where this algorithm is used for optimizing single model U-shaped assembly line.

There are many application possibilities for solving assembly line multi-objective optimization problem using genetic algorithms. Examples of their using in this area with detail description of their functionality can be found in [14].[15] is using genetic algorithm in dealing with assembly sequence planning problem. Solution for defining and two methods of pruning optimal Pareto set are mentioned in [16].

Variation of production rates and number of assembly line setups are optimized simultaneously with using multi-objective genetic algorithm approach in[17].

Using metaheuristic method of tabu search for solving simple assembly line balancing problem is described in [18].

B. *Mathematical approach*

From the 70's, when first vector optimization tasks were solved, numbers of methods dealing with this issue were created. All these methods assume that there is a definition of more than one objective function, as well as list of equality and inequality constraints defining feasible solution space. Mathematical methods dealing with this manner can be divided according to [7] into three main groups:

- methods defining the set of non improving elements ,
- compromising methods
- methods of hierarchical criteria sequence.

Methods defining the set of non improving elements

There is no hierarchy of criteria in this group of methods, every criterion is equally important. Task of vector optimization can be defined as trying to minimize the vector $\boldsymbol{J}(\boldsymbol{x}) - \boldsymbol{J}(\boldsymbol{x_\alpha})$, where $\boldsymbol{x_\alpha}$ is optimal values vector of variables $\boldsymbol{x}$ according to defined criteria.

a) Quadratic norm

The most common criterion used in this norm is minimal sum of quadratic variance of objective functions $J_\alpha(\boldsymbol{x})$ for random $\boldsymbol{x} \in \{X\}$ from objective function $J_\alpha(\boldsymbol{x_\alpha})$ for vector of ideal values of chosen criteria $\boldsymbol{x_\alpha} = (x_{1\alpha}, x_{2\alpha}, \dots, x_{n\alpha})$, $\alpha = 1,2, \dots, k$;

$$R(x) = \sum_{\alpha=1}^{k} (\boldsymbol{J_\alpha}(\boldsymbol{x}) - \boldsymbol{J_\alpha}(\boldsymbol{x_\alpha}))^2$$
$$opt\ R(x) = \min_{x \in \{X\}} \sum_{\alpha=1}^{k} (\boldsymbol{J_\alpha}(\boldsymbol{x}) - \boldsymbol{J_\alpha}(\boldsymbol{x_\alpha}))^2 \tag{6}$$

Values of this norm are usually divided by optimal value. Reason for this is non-dimensional solution value. Calculation formula then looks like:

$$R(x) = \sum_{\alpha=1}^{k} \frac{(\boldsymbol{J_\alpha}(\boldsymbol{x}) - \boldsymbol{J_\alpha}(\boldsymbol{x_\alpha}))^2}{\boldsymbol{J_\alpha}(\boldsymbol{x_\alpha})^2}$$

$$opt\ R(x) = \min_{x \in \{X\}} \sum_{\alpha=1}^{k} \frac{(\boldsymbol{J_\alpha}(\boldsymbol{x}) - \boldsymbol{J_\alpha}(\boldsymbol{x_\alpha}))^2}{\boldsymbol{J_\alpha}(\boldsymbol{x_\alpha})^2}. \tag{7}$$

b) Linear norm

It represents minimal sum of linear variance of objective functions $(\boldsymbol{J_\alpha}(\boldsymbol{x}))$ from optimal values of objective functions $\boldsymbol{J_\alpha}(\boldsymbol{x_\alpha})$. Optimal value of functions $R(x)$ can be calculated using formulas:

$$R(x) = \left| \sum_{\alpha=1}^{k} (\boldsymbol{J_\alpha}(\boldsymbol{x}) - \boldsymbol{J_\alpha}(\boldsymbol{x_\alpha}) \right|,$$

$$opt\ R(x) = \min_{x \in \{X\}} \left(\sum_{\alpha=1}^{k} (\boldsymbol{J_\alpha}(\boldsymbol{x}) - \boldsymbol{J_\alpha}(\boldsymbol{x_\alpha}) \right). \tag{8}$$

c) Generalized norm

Objective functions of this norm is

$$R_L(x) = \sum_{\alpha=1}^{k} \left\{ \left((\boldsymbol{J_\alpha}(\boldsymbol{x}) - \boldsymbol{J_\alpha}(\boldsymbol{x_\alpha}) \right)^L \right\}^{\frac{1}{L}}; L \geq 1. \tag{9}$$

For $L = 1$ this functions corresponds to linear norm, for $L = 2$ it corresponds quadratic norm and for $L = \infty$ it is

$$R_\infty(x) = \max_\beta\{(\boldsymbol{J_\alpha}(\boldsymbol{x}) - \boldsymbol{J_\alpha}(\boldsymbol{x_\alpha}); = 1,2, \dots, n\}. \tag{10}$$

All these norms can be refined by multiplying objective function elements with appropriate coefficients. Searched point $\boldsymbol{x}^* \in \{X\}$ is called non improving in space $\{X\}$ regarding to functional $\boldsymbol{J}(\boldsymbol{x})$, if there is no point $\widetilde{\boldsymbol{x}}$ in this space, for which is valid $(\boldsymbol{J_\alpha}(\widetilde{\boldsymbol{x}}) \leq \boldsymbol{J_\alpha}(\boldsymbol{x}^*), \alpha = 1,2, \dots, k.$

<u>*Compromising methods*</u>

These methods are based on defining more strict constrains or adding another constrains to objective functions. Compromise means finding optimal solution by minimizing value of formula $\beta_1 J_1(\boldsymbol{x}) + \beta_2 J_2(\boldsymbol{x}) + \cdots + \beta_k J_k(\boldsymbol{x})$, where β_1 to β_k are carefully chosen importance coefficients. Their values are recommended to be chosen as follows: $\beta_1 = 1/J_{10}, \beta_2 = 1/J_{10}, \beta_k = 1/J_{k0}$, where J_{k0} are values of objective functions counted in optimization only via selected criterion.

These methods are used when it is possible to define importance of every criterion before starting optimization process, or when additional information about criterion importance is found out during optimization process. Main idea is in defining importance of every scalar criterion. This can affect the result of vector optimization (importance of scalar criterion will be labeled as λ). Most common methods from this group are optimization of weighted sum of scalar criteria and weighted sum of variance vector.

d) Optimization of weighted sum of scalar criteria

Function for this norm is defined by formula

$$opt\ R(x) = opt \sum_{\alpha=1}^{k} \boldsymbol{\lambda_\alpha J_\alpha}(\boldsymbol{x}). \tag{11}$$

Optimal solution is represented by maximum or minimum of weighted sum of scalar criteria, depending on character of problem being solved.

e) Weighted sum of variance vector

In this norm function $R(x)$ is represented by formula

$$R(x) = \sum_{\alpha=1}^{k} \boldsymbol{\lambda_\alpha} * (\boldsymbol{J_\alpha}(\boldsymbol{x}) - \boldsymbol{J_\alpha}(\boldsymbol{x_\alpha}). \tag{12}$$

Optimal solution is always represented by minimum value of function $R(x)$, because this norm is using variation from ideal values.

$$opt\ R(x) = \min_{x \in \{X\}} \left(\sum_{\alpha=1}^{k} \boldsymbol{\lambda_\alpha} * (\boldsymbol{J_\alpha} - \boldsymbol{J_\alpha}(\boldsymbol{x_\alpha}) \right). \tag{13}$$

It is also possible to use quadratic variant of this formula (weighted quadratic norm).[7]

<u>*Methods of hierarchical criteria sequence*</u>

In this qroup of methods, hierarchical sequence of criteria is created, so there is one superior criterion chosen from list of criteria, and all others are inferior to it.

Methods mentioned in this paper are not the only one used in solving multi-objective optimization problems, other methods can be found in [9] and [19].

III. Multi-objective assembly line optimization

First step in solving multi-objective optimization of assembly lines is creating a detail model of system. Assembly lines can be modeled by various means. During my work will be used Petri nets and Stateflow diagrams. Assembly lines modeling using Petri nets, as well as other possibilities for modeling assembly lines can be found in [20].

Created model must truly display real assembly line, for example follow precedence constraints of assembly line. This model can be also helpful in finding critical spots in production process, as well as defining optimization criteria:

- maximizing profit level, resp. reducing costs,
- maximizing reliability and safety,
- maximizing efficiency resp. minimizing overload,
- minimizing manual interventions,
- minimizing of production time.

Assembly line model can be also source for defining constraints for this multi-objective optimization tasks, f.e.:

- maximum level of costs,
- maximum time in one work shift,
- minimum level of needed sources etc.

All criteria and constraints should be defined by mathematical equations, representing objective functions. Once there are defined, a multi-objective optimization task is built and there is a place to find ways of solving it.

Focus of my dissertation thesis is to solve multi-objective optimization problem using mathematical approach. In my previous work, application able to solve vector optimization tasks was created in MATLAB. Present version of this application can be used for solving vector optimization problems with 2 objective functions. One of them can have either quadratic or linear form, the other one must be linear. Also all constraints have to be defined by linear equations or inequations. Solving sample tasks from production and economical sector using created application is listed in [21]. Algorithm solving mentioned vector optimization tasks is displayed in Fig. 4.

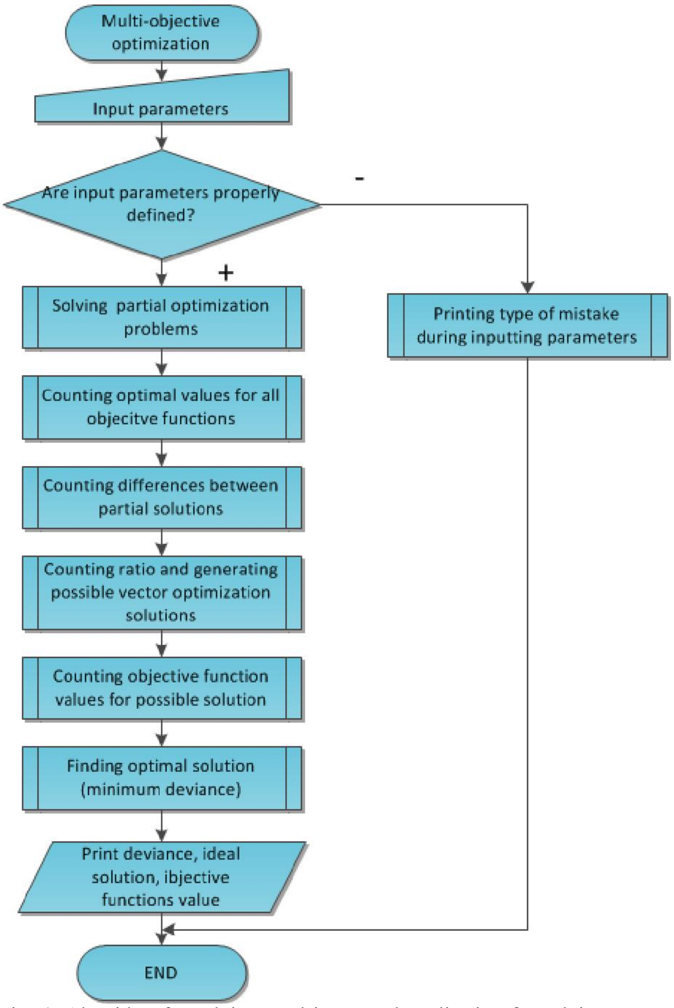

Fig. 4. Algorithm for solving used in created application for solving vector optimization problem

Application has also its own graphic user interface (GUI) designed for simple and intuitive inputting parameters, as well as for clear and easy-to-read displaying outputs. GUI of created application is shown in Fig. 5.

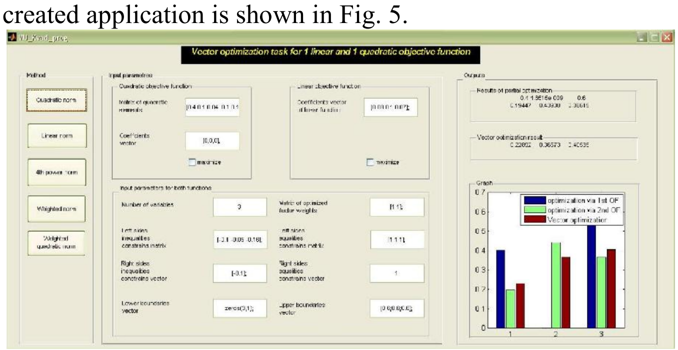

Fig. 5 Application created for solving vector optimization tasks

According to the fact, that this application can solve only limited number of tasks, my goal in my future work is to make it more complex and usable in solving optimization tasks described by other, more complicated objective functions and constraints.

Another task, which has to be solved in my future work on dissertation thesis is creation of detail model of optimized production line. For this purpose Stateflow diagrams will be used. Main advantage of this form of assembly line modeling is simple, but complex syntax able to model large variety of processes executed in production lines.

At the basis of this model, mathematical description of defined optimizing parameters will be created.

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