

# Design of Adaptive Control Algorithm of Mechatronic System Ball&Plate

<sup>1</sup>Matej Oravec, <sup>2</sup>Anna Jadlovská, <sup>3</sup>Dana Novisedláková

Department of Cybernetics and Artificial Intelligence, Faculty of Electrical Engineering and Informatics, Technical University of Košice, Slovak Republic

<sup>1</sup>matej.oravec@tuke.sk, <sup>2</sup>anna.jadlovska@tuke.sk,  
<sup>3</sup>dana.novisedlakova@student.tuke.sk

**Abstract** — This paper describes the design of an adaptive control algorithm and its application for simulation of a model of the B&P mechanical system. In this paper is explained the mathematical description of the B&P system and the design of the adaptive control algorithm with elements of the self-tuning. The adaptive control used in correct control structure is very important part of the paper. A recursive least squares method is a part of using algorithm for identification of the system parameters. System description assumed ARX model. Control design is based on pole placement method and identified system parameters. The proposed adaptive control algorithm was verified at the nonlinear Ball and Plate model simulation using the control structure of Matlab/Simulink environment.

**Keywords** – Algorithm of adaptive control, recursive least square method, pole placement, mechatronic system Ball&Plate, regressive model ARX

## I. INTRODUCTION

The Ball and Plate is a training model used by many educational institutions in the world. The Center of Modern Control Techniques and Industrial Informatics at the Technical University of Košice own two such models. The first Humusoft CE 151 on which the classic control algorithms are tested, namely the algorithm of Naslin, Graham-Lathrop, and Butterworth, which are described in papers [4] and [5]. Application of optimal control algorithms to the Ball and Plate Humusoft CE 151 was unsuccessful. This fact was motivation to test different types of algorithms on the second Ball and Plate model. This model called B&P\_KYB, which simulation model is described in this paper, is a documented in papers [6] and [7] and on which classical control algorithms of Naslin, Butterworth and Graham-Lathrop has been designed and validated. Unlike the first system CE 151, on the model B&P\_KYB the state and predictive control method were applied, where the results were listed in papers [5] and [7]. The functionality of predictive control algorithms used on the B&P\_KYB model, led us to the idea to try and apply the adaptive control algorithm on a nonlinear simulation model to verify it on a real system.

The adaptive control methods are being designed mainly in a discrete form and are suitable for industrial applications. From literature sources it is known that the adaptive control algorithm with self-tuning character was implemented into control of the hydraulic system of two and three containers [1], the mixing of the reactor [2] and for controlling the temperature of a thermal analyzer [3]. This paper will focus on proposals for indirect adaptive control with auto tuning features for managing the Ball and Plate simulation model.

## II. ADAPTIVE CONTROL OF MECHATRONICAL SYSTEMS

The adaptive control systems with auto tuning elements belong to the group of indirect adaptive systems. The control structure of adaptive systems contains two more modules than classic control structures, Fig. 1. The first module used for on-line identification (parameter estimation system) and the second one for the control computation (calculation of controller coefficients and law control). Sample period is  $T_s = 0,05s$ . Algorithm of self-tuning controller for the third order system with the third order polynomial controller is described in steps:

1. Choice of require poles  $z_1, z_2, z_3, z_4, z_5, z_6$
2. On-line identification, which result is vector with parameters of system in the form:  
 $\theta^T(k) = [a_1(k), a_2(k), a_3(k), b_1(k), b_2(k), b_3(k)]$
3. Calculation of controller coefficients  $p_1, p_2, p_3$  and  $q_0, q_1, q_2, q_3$
4. Calculation of control law  $u(k)$

5. Calculation the position of the ball  $y(k)$
6. If the conditions  $t < T_{fin}$  and  $|\theta(k) - \theta(k-1)| < \varepsilon$  ( $\varepsilon$  is precision) are true, then go to step 2, else go to step 7
7. If the condition  $t < T_{fin}$  is true, then go to step 8, else go to step 10
8. Calculation of control law  $u(k)$
9. Calculation the position of the ball  $y(k)$  and go to step 7
10. Algorithm finished

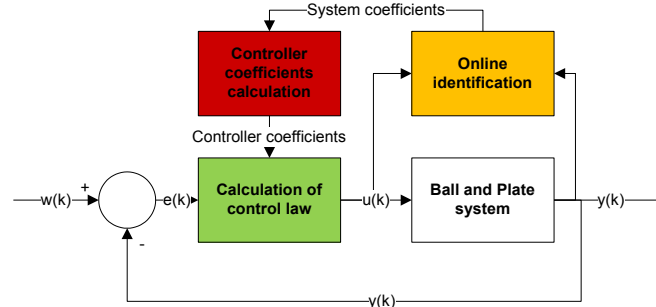


Fig. 1 The control structure of self-tuning controller

#### A. The B&P simulation model

The laboratory model B&P\_KYB is an educational model, which is located in the laboratory of mechatronic systems V142. The model B&P\_KYB contains servomotors whose mathematical model is unknown and therefore it is necessary to use experimental identification to obtain their description. Connection with the computer model is carried out using a single-chip microcomputer. For determination of the position of the ball a simple camera is used [8].

Further, we will deal with the creation of the simulation model, which will follow the real B&P\_KYB system.

A non-linear dynamic system of the Ball and Plate (B&P) represents a mechanical system, which can be divided into 4 sub-systems (Fig. 2). Ball and Plate contains sub-system of the servomotor for the axis  $x$  and axis  $y$ , and a subsystem of Ball and Plate in a direction of axis  $x$  and axis  $y$ .

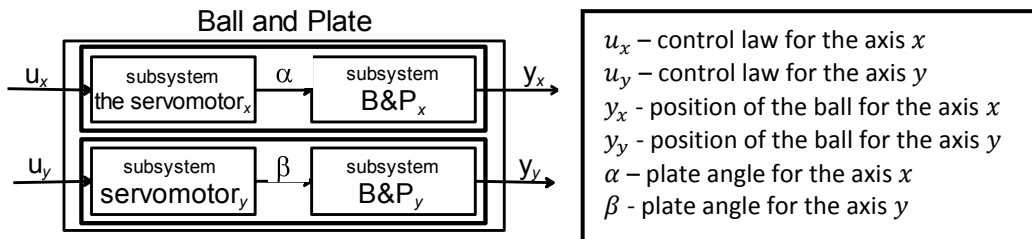


Fig. 2 Ball and Plate B&P\_Kyb system decomposition

The mathematical description of the Ball and Plate subsystem is derived from the Euler-Lagrange differential equations and is in the form:

$$\begin{aligned} \text{axis } x: & \quad \left(\frac{J}{r^2} + m\right) \ddot{x}(t) - m \cdot \dot{\alpha}^2(t) \cdot x(t) - m \cdot g \cdot \sin\alpha(t) = 0 \\ \text{axis } y: & \quad \left(\frac{J}{r^2} + m\right) \ddot{y}(t) - m \cdot \dot{\beta}^2(t) \cdot y(t) - m \cdot g \cdot \sin\beta(t) = 0 \end{aligned}$$

The moment of the ball inertia is  $J$ , the radius of the ball  $r$ , positions of the ball are  $x$ ,  $y$  the mass of the ball is  $m$  and the gravitation acceleration is  $g$ .

By approximation we can replace the used subsystems of servomotors by a transfer function [8], which is in the form:

$$\text{axis } x: \quad F_{servo_x}(s) = \frac{K_u}{T_a s + 1}$$

$$\text{axis } y: \quad F_{servo_y}(s) = \frac{K_u}{T_\beta s + 1}$$

where parameters  $K_u, T_a, T_\beta$  are listed in paper [8].

On the basis of known system parameters [8] a non-linear dynamic simulation model of the B&P\_KYB in the Simulink environment has been created.

The results obtained by simulation, where for B&P model control the continuous control algorithms of Naslin, Graham-Lathrop and Butterworth methods were used, and the discrete control algorithms based on pole placement were compared with the results obtained from the real model.

#### B. Method for on-line identification with used regression model ARX

Method for on-line identification is used for the parameters identification of the system, that are used for compute parameters of control.

Algorithm for on-line identification use recursive least square method. Main goal of the on-line identification is vector of parameters for the system described by input-output form:

$$\theta^T(k) = [a_1(k), a_2(k), a_3(k), b_1(k), b_2(k), b_3(k)]$$

Regression vector of system input and output previous values, is in form:

$$\phi(k-1) = [y(k-1), y(k-2), y(k-3), u(k-1), u(k-2), u(k-3)]$$

Calculation of system parameters identification is realized:

1. Calculation prediction error:

$$e(k) = y(k) - \hat{\theta}^T(k-1)\phi(k-1)$$

2. Calculation vector of new system parameters:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{C(k-1)\phi(k-1)}{\varphi + \phi^T(k-1)C(k-1)\phi(k-1)} e(k)$$

3. Calculation of covariance matrix:

$$C(k) = \frac{1}{\varphi} \left( C(k-1) - \frac{C(k-1)\phi(k-1)\phi^T(k-1)C(k-1)}{\varphi + \phi^T(k-1)C(k-1)\phi(k-1)} \right)$$

Identified system parameters are used for compute of control parameters with pole-placement method.

#### C. Verification of the designed adaptive control algorithm for the B&P\_KYB nonlinear model

For performing synthesis with pole placement method we assume simulation model will 3. orders in the form:

$$F_S(z) = \frac{B(z)}{A(z)} = \frac{b_1z^{-1} + b_2z^{-2} + b_3z^{-3}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3}}$$

On the basis of the system order, we assume transfer function of controller in the form with the same order:

$$F_R(z) = \frac{Q(z)}{P(z)} = \frac{q_0 + q_1z^{-1} + q_2z^{-2} + q_3z^{-3}}{1 + p_1z^{-1} + p_2z^{-2} + p_3z^{-3}}$$

Calculation of controller parameters is realized follows:

1. Calculation characteristic polynomial consists of system parameters and assumed form of controller:

$$N(z) = P(z).A(z) + B(z).Q(z)$$

2. Create polynomial of require poles  $z_1, z_2, z_3, z_4, z_5, z_6$ :

$$N_R(z) = (z - z_1)(z - z_2) \dots (z - z_6)$$

Comparing characteristic polynomial and polynomial of required poles at the same of powers of the variable  $z$  is created 6 equation and 7 unknown parameters.

3. Of the theoretical nature of this method:  $\sum_{i=1}^n p_i = -1$

This calculation of controller parameters based on on-line identification of system parameters is realized in enviroment MATLAB as subroutine. It is implemented in block in enviroment Simulink with interpreted MATLAB Function.

On the basis of the control parameters and the values of the error (difference between reference trajectory and system output) and control law from previous iterations. Control law is computed from:

$$u(k) = q_0 \cdot e(k) + q_1 \cdot e(k-1) + q_2 \cdot e(k-2) + q_3 \cdot e(k-3) + p_1 \cdot u(k-1) + p_2 \cdot u(k-2) + p_3 \cdot u(k-3)$$

On-line identification of system parameters and controller parameters calculation is created as subroutine and it is implemented in block of the S-function in simulations environment Simulink.

#### D. Verification of designed adaptive control for nonlinear system B&P\_KYB

For verification of the solution a simulation scheme of the adaptive controller in the Simulink environment was created, which contains the blocks described above. The adaptive controller was applied on the B&P\_KYB mechatronic system. For the simulation the initial parameters of the system were chosen, which were acquired by experimental identification (only once) of the B&P system, the forgetting factor  $\varphi$  was set to 0.99, the initial values of the regression vector  $\phi$  were zeroes and for the covariance matrix  $C$  were  $10^{-6}$ . Executing the simulation the presented results were achieved, illustrated in Fig. 3, Fig. 5, Fig. 6.

In Fig. 3, the observed trajectories of the Ball in the direction of axis  $x$  (up) and in the direction of axis  $y$  (down) are shown, which are result of the adaptive control algorithm.

In Fig. 4, the process of parameter development of the B&P\_KYB simulation system is shown, which was obtained by the on-line identification algorithm.

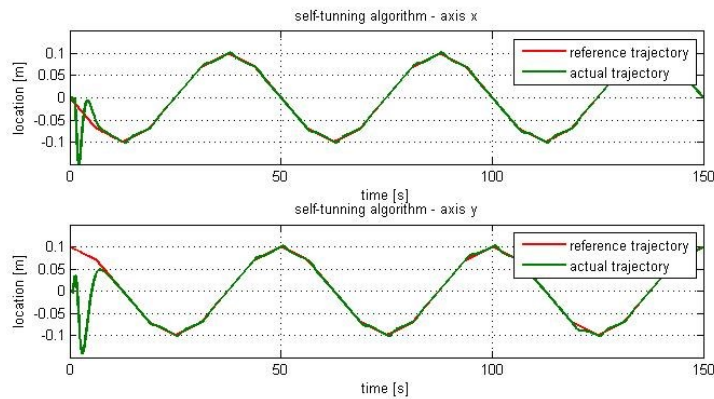


Fig. 3 Tracking the reference trajectory – adaptive control

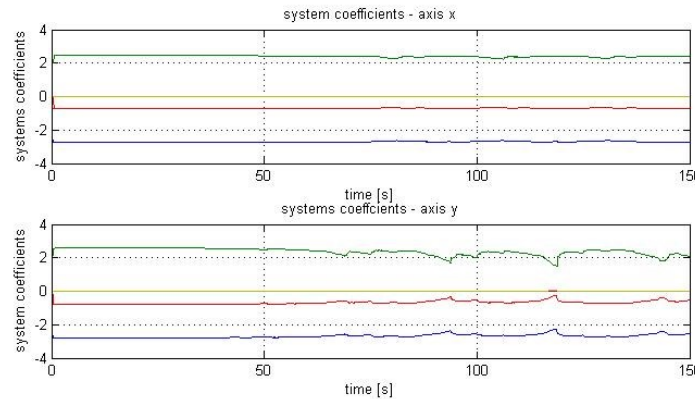


Fig. 4 Development parameters for axis  $x$  and axis  $y$

For comparison, in Fig. 5 are shown the obtained trajectories obtained by controlling the B&P\_KYB simulation model using a classic controller based on chosen poles, which is designed by the input-output model of the B&P\_KYB system.

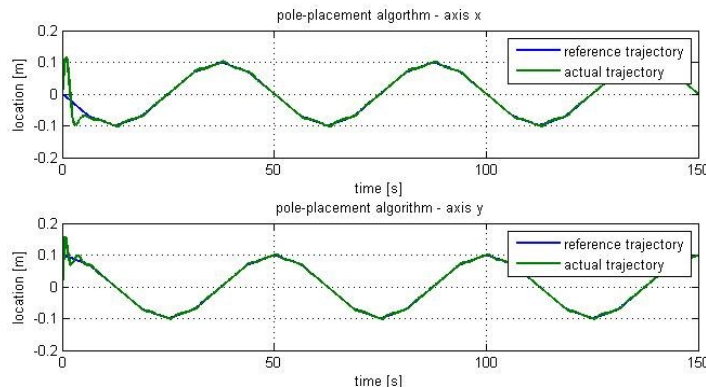


Fig. 5 Tracking the reference trajectory – polynomial controller based on pole-placement

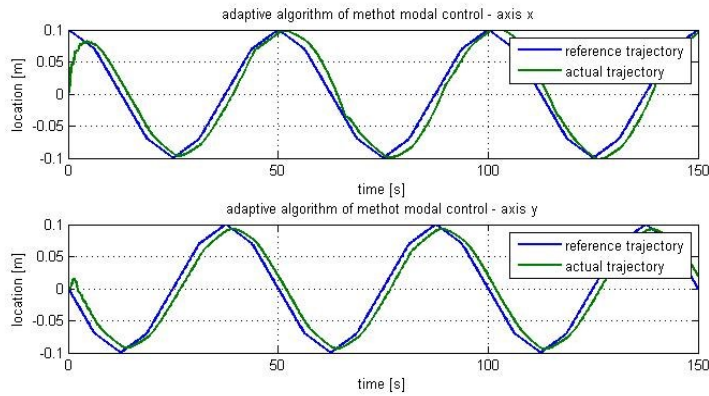


Fig. 6 Tracking the reference trajectory – Modal control

Results of modal control can be found in Fig. 6. The differences between the control along the axis  $x$  and the axis  $y$  are caused by using different servomechanisms with different dynamic properties.

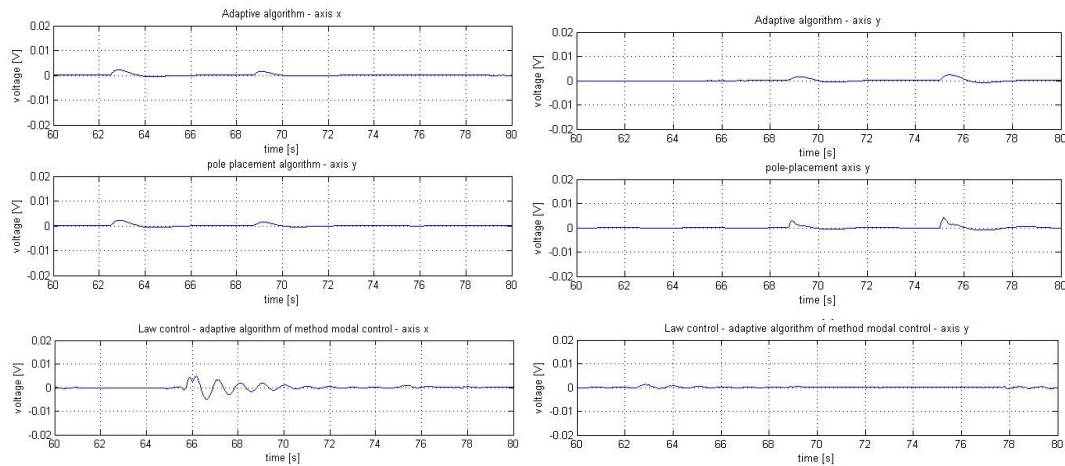


Fig. 7 Time response of the control input - on the left side for axis  $x$  and on the right side for axis  $y$ , from up to down: adaptive control algorithm, pole placement algorithm, modal control algorithm

Fig. 7 contains the details of law controls obtained by the described control methods. For all three of these methods, the details are selected from the 60th to 80th second of the solution, while the entire simulation lasted 150 seconds. From these trajectories, it is clear that the use of adaptive control methods the law control is much more significant.

### III. CONCLUSION

The papers proposes an adaptive control algorithm based on the pole placement method and its application on the educational B&P\_KYB simulation model. Results of the control of non-linear B&P\_KYB simulation model with the usage of the adaptive control algorithm are presented at this paper. Very important fact acquired from the simulations, that it is possible to use this type of control algorithm for the B&P\_KYB model, although it is a faster dynamic system. The results were compared with the results using control system based on the pole placement method without adaptation.

### ACKNOWLEDGMENT

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