Optimal control of the mechatronicalal laboratory model B&P_KYB

Matej ORAVEC, Anna JADLOVSKÁ
Department of cybernetics and artificial intelligence, Faculty of Electrical Engineering and Informatics, Technical university in Košice, Slovak republic

oravec.matej@atknet.sk, anna.jadlovska@tuke.sk

***Abstract* — This article describes new mechatronical laboratory model B&P_KYB. This model is specific in construction and used construction parts. Article is focused on the modeling of mechatronical laboratory model B&P_KYB. Also it is focused on creation, analysis and validation of simulation model B&P_KYB. For the trajectory tracking, optimal LQ control algorithm with an integrator in discrete form has been implemented. This algorithm is based on state model of mechatronical system B&P_KYB. For optimal LQ control algorithm with an integrator in discrete form a correct scheme have to be chosen. Optimal LQ control algorithm with an integrator in discrete form is verified on real and mechatronical model B&P_KYB. Article presents a part of several results stated in thesis [5].**

***Key words* — Ball and plate, optimal LQ control, optimal LQ control with an integrator in discrete form**

I. Introduction

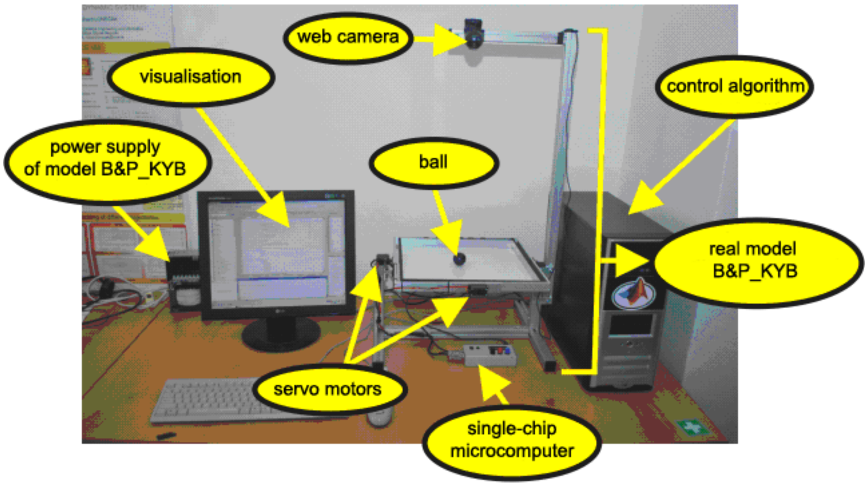
Mechatronical laboratory model B&P_KYB is model of ball on plate, it exists a lot of different variants of this kind of mechatronical model (e. g. model of Humusoft CE151 - [7], [8], [1]). Model B&P_KYB has a lot of specific components, which are unique. For the plate sloping are used servo motors from hobby models, but those have disadvantage, because their parameters aren't known and use of the experimental methods of identification is necessary (e.g. model of Humusoft CE151 used step motors - [7], [8]). The connection of servomotors with the control PC is being done by using single-chip microcomputer, what is different to another kinds of models, which are using more expensive PLC or Lab cards. The next difference is the use of simple web camera (frames per second have to be more than 30). The advantage of model B&P_KYB is the possibility to use any web camera with similar parameters , but it is necessary to implement own algorithm for image processing (e.g. model of Humusoft CE151 used Matlab Image processing toolbox).

In fact, model B&P_KYB is different model than another ball and plate models (e. g. Humusoft CE151 - [7], [8]), so one of purpose of this article is to present the creation process of simulation model of B&P_KYB. Parameters of simulation model of B&P_KYB can be used for design and verification of control algorithms (e. g. PSD, polynomial, optimal state-space LQ, predictive) before they will be used on a real model. In this article is presented the algorithm of optimal state - space LQ control with an integrator in discrete form (an integrator in discrete form). Another types of control algorithms of the mechatronical model B&P_KYB are presented in [5].

II. Laboratory model of B&P_KYB

A. *Description of the laboratory model B&P_KYB*

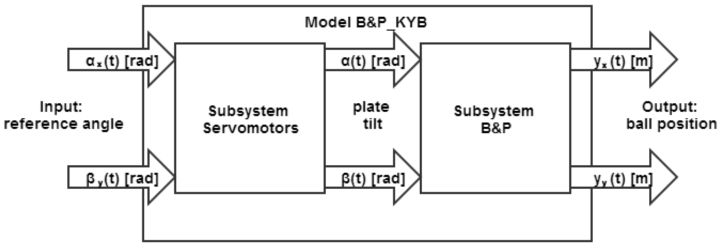
The laboratory mechatronical model B&P_KYB (Im. 1) is a real model, which is located in Laboratory of mechatronical systems V142, Department of Cybernetics and artifical inteligence, FEEI, TU ([6]). Model consists of two servo motors, plate, ball, web camera and single-chip microcomputer. Single-chip microcomputer is interface for control of servo motors from PC. Servo motors are tilting a plate and ball is moving on a plate. Web camera is being fixed above the plate and capturing the position of ball. For calculating of ball position on the plate, an algorithm for image processing in language *C#* using libraries of *emguCV*.



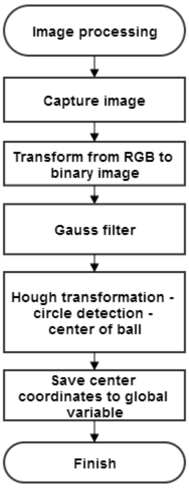
Obr. 1 Mechatronical laboratory model B&P_KYB – Laboratory of mechatronicalal systems V142 ([6])

The schema of laboratory model B&P_KYB assumes that the system is divided to two subsystems. First subsystem consists of servo motors, second subsystem consists of ball on plate (Im. 2). The next assumption, is that this laboratory model is not being taken as MIMO system, but as twi SISO systems.. Based on this fact, the system can be divided to two separate system, one for direction of axis *x* and second for direction of axis *y*.

Input of subsystem *Servo motors* for direction of axis *x* (*y*) is reference angle $\alpha_x(t)$ ($\beta_y(t)$), output is angle $\alpha(t)$ ($\beta(t)$). Angle $\alpha(t)$ ($\beta(t)$) is input for subsystem *B&P* for direction of axis *x* (*y*), output of this subsystem is *x* (*y*) direction of ball position $y_x(t)$ ($y_y(t)$).



Im. 2 Scheme of mechatronical laboratory model B&P_KYB



Im. 3 Image processing algorithm of mechatronical laboratory model B&P_KYB

How has been described, ball position $y_x(t)$, $y_y(t)$ is being captured by web camera above the plate. For the use of web camera it has been important to design and implement algorithm for image processing (Im. 3), which output is ball position $y_x(t)$, $y_y(t)$. Servo motors are communicating with PC with use of single-chip microcomputer. Communication is realized by

RS232 interface protocol. Another real mechatronical models are using for communication Lab card (not single-chip microcomputer) with Matlab RealTime toolbox, e.g. Humusoft CE151 ([6], [8]).

B. Simulation model of B&P_KYB

Target of simulation model is exactly described real laboratory model B&P_KYB. Simulation model of B&P_KYB should be used in situation, when real model is not available or control algorithm could broke real laboratory model. Simulation model is based on mathematical description of real model by differential equations with using physical laws.

Model B&P_KYB consist of two subsystems, subsystem *Servo motors* and subsystem *B&P*. Subsystem *Servo motors* describes linear differential equations:

$$\dot{\alpha}(t) = \frac{K_x}{T_{sx}} \cdot (\alpha_x(t) - \alpha(t)), \tag{1}$$

$$\dot{\beta}(t) = \frac{K_y}{T_{sy}} \cdot (\beta_y(t) - \beta(t)) \cdot \tag{2}$$

where K_x, K_y are gain coefficients of servo motors and T_{sx}, T_{sy} are time constant of servo motors. Parameters of servo motors in linear differential equations are obtained by identification from step response (method of surface [4]).

Subsystem *B&P* describes two nonlinear differential equation. For their derivation are used Lagrange equations of second of second kind. Parameters of model B&P_KYB are ball radius r [m], ball mass m [kg] and plate length L [m]. Subsystem *B&P* describe equations in direction of axis x and y (more in [5]):

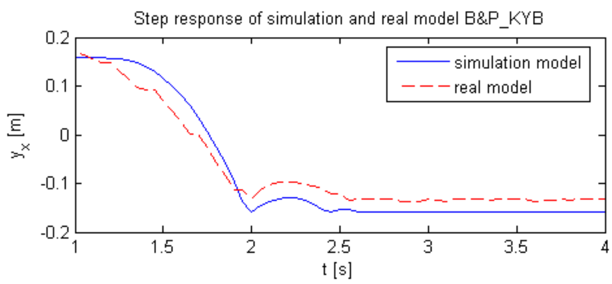
$$\left(\frac{J}{r^2} + m\right) \cdot \ddot{y}_x(t) - m \cdot g \cdot \sin\alpha(t) = 0 \quad \rightarrow \quad \ddot{y}_x(t) = \frac{5}{7} g \sin\alpha(t), \tag{3}$$

$$\left(\frac{J}{r^2} + m\right) \cdot \ddot{y}_y(t) - m \cdot g \cdot \sin\beta(t) = 0 \quad \rightarrow \quad \ddot{y}_y(t) = \frac{5}{7} g \sin\beta(t), \tag{4}$$

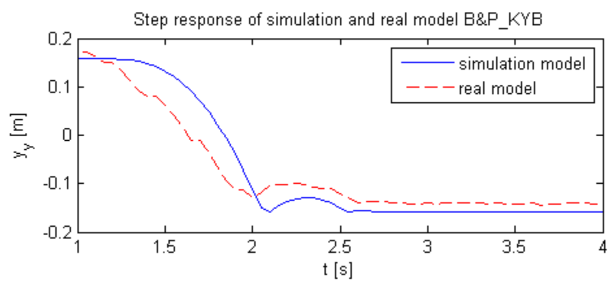
where for moment of inertia: $J = \frac{2}{5} mr^2$.

The plate of mechatronical model B&P_KYB, where the ball is moving, has the form of square of dimensions <-L/2, L/2>. In the simulation model B&P_KYB those limitations are being incorporated (Im. 3, Im. 4).

For verification of simulation model with a real laboratory model was used the same step input signal. Verification is illustrated in Im. 3 and Im. 4, which shows, that response are nearly same and the simulation model is suitable replacement of real model for simulation and control algorithm testing. This simulation model is in form of Grey box model.



Im. 3 Comparison of simulation and real model – direction of axis *x*



Im. 4 Comparison of simulation and real model – direction of axis *y*

III. Optimal LQ control of real and simulation model B&P_KYB

For the control B&P_KYB laboratory model it is possible to use many different control algorithms, e.g. PSD, polynomial, predictive (design of some control algorithm and their verification on simulation and real model are shown in [5]). One of control algorithm, which can be used for simulation or real model B&P_KYB, is optimal LQ control algorithm. This type of optimal control is used for feedback control if the target is the control to equilibrium or steady state (with feed-forward). If the target of control is reference trajectory tracking, it is necessary to implement the algorithm of optimal LQ control with an integrator in discrete form. For design of optimal LQ control algorithm with an integrator in discrete form is important to define state space description of model B&P_KYB.

A. State space model B&P_KYB

For the creation of state space model it is important to define state variables. Vector of state variables for direction of axis *x* is:

$$\bar{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} y_x(t) \\ \dot{y}_x(t) \\ \alpha(t) \end{bmatrix} = \begin{bmatrix} position\ [m] \\ velocity\ [m \cdot s^{-1}] \\ angle\ [rad] \end{bmatrix} \tag{5}$$

Input vector has form:

$$\bar{u}(t) = [\alpha_x(t)] \tag{6}$$

From equation (1) and (3) is able to define substitution in canonical form (direction of axis *x*):

$$y_x(t) = x_1(t)$$

$$f_1: \quad \dot{y}_x(t) = x_2(t) \tag{7}$$

$$f_2: \quad \dot{x}_2(t) = \frac{5}{7} g \sin x_3(t) \tag{8}$$

$$f_3: \quad \dot{x}_3(t) = \frac{K_x}{T_{sx}} \cdot (\alpha_x(t) - x_3(t)) \tag{9}$$

For linearization of system B&P_KYB in direction of axis *x* the reference angle is $\alpha_x(t) = 0$, ball position $y_x(t) = 0$ and velocity $\dot{y}_x(t) = 0$. Linearization of system is realized in equilibrium point, which directions are $x_{XPB} = [x_1 = 0 \quad x_2 = 0 \quad x_3 = 0]$.

Continuous state space of model B&P_KYB (direction of axis *x*):

$$\begin{aligned} \dot{x}_x(t) &= A_x x_x(t) + B_x u_x(t) \\ y_x(t) &= C_x^T x_x(t) \end{aligned} \tag{10}$$

System matrix could be defined by Jacoby matrix:

$$A_x = \left.\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix}\right|_{x_{XPB}} = \left.\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & k_g g \\ 0 & 0 & -\frac{K_x}{T_{sx}} \end{bmatrix}\right|_{x_{XPB}}, \quad k_g = \frac{5}{7} \tag{11}$$

Input matrix is:

$$B_x = \left.\begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \\ \frac{\partial f_3}{\partial u} \end{bmatrix}\right|_{x_{XPB}} = \left.\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right|_{x_{XPB}} \tag{12}$$

Process to define state space of system for direction of axis *y* is the same like for axis *x*. Output matrixes for both directions are: $C_x^T = C_y^T = [1 \quad 0 \quad 0]$.

For mechatronical model B&P_KYB the algorithm of optimal control must be in discrete form. State space equations of model in discrete form are following:

$$\begin{aligned} x_x(k+1) &= F_x x_x(k) + G_x u_x(k) \\ y_x(k) &= C_x^T x_x(k) \end{aligned}, \tag{13}$$

$$\begin{aligned} x_y(k+1) &= F_y x_y(k) + G_y u_y(k) \\ y_y(k) &= C_y^T x_y(k) \end{aligned}, \tag{14}$$

where system matrixes F_x, F_y are (3x3) and input matrixes G_x, G_y are (3x1). Output matrixes are same like in continuous time state space matrixes.

Sample period is T_s = 0,05 s.

B. Implementation of optimal LQ algorithm with an integrator in discrete form

Implementation of optimal LQ algorithm assumed, that quadratic functional has been minimized:

$$\begin{aligned} J_{LQ} &= x^T(M)\,Q\,x(M) + \\ &+ \sum_{i=0}^{M-1}(x^T(i)\,Q\,x(i) + x^T(i)\,S\,u(i) + u^T(i)\,S^T\,x(i) + u^T(i)\,R\,u(i)) \end{aligned}, \tag{15}$$

where M is finite horizont. Functional J_{LQ} has been minimized with respect to limitations, which actually are discrete state equations. Matrix Q of the functional is weight matrix of state variables (positive semidefinite), matrix R is input weight matrix (positive definite) and matrix S is zero matrix (dimension (3x1)). Values of these matrices are optional, but have to suit those mentioned conditions. To determine the state feedback gain k_x it is necessary solved discrete Riccati algebraic equation ([2]):

$$P = Q + F^T PF - [S + F^T PG][R + G^T PG]^{-1}[S + F^T PG]^T\,, \tag{16}$$

and equation for state feedback gain is:

$$k_x = [R + G^T PG]^{-1}[S + FPG]^T\,. \tag{17}$$

This solution of state feedback amplification could be used only for target of control to equilibrium.

For reference trajectory tracking it's necessary to add discrete form of an integrator of control deviation S_u. Detail solution for an integrator in discrete form could be find in [3], for the calculation of an integrator in discrete form in step *k+1* applies:

$$\begin{aligned} S_u(k+1) &= S_u(k) + K_2 e(k) = S_u(k) + K_2(w(k) - y(k)) = \\ &= S_u(k) + K_2(w(k) - Cx(k)) \end{aligned}, \tag{18}$$

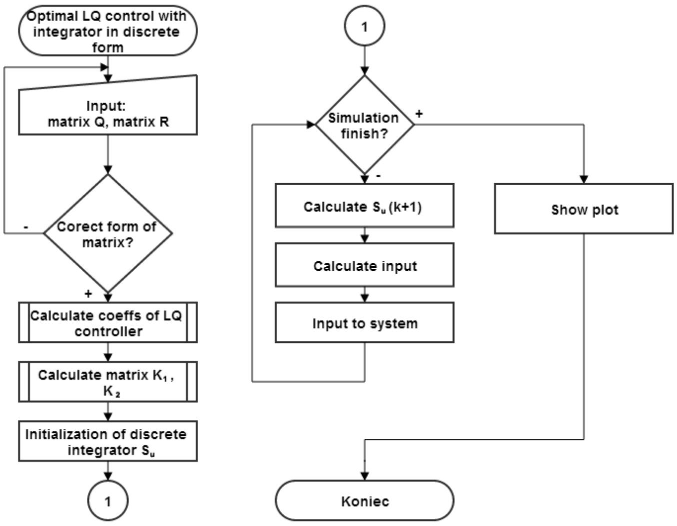
for new input:

$$u(k+1) = -K_1 x(k+1) + S_u(k+1) + K_2 w(k)\,, \tag{19}$$

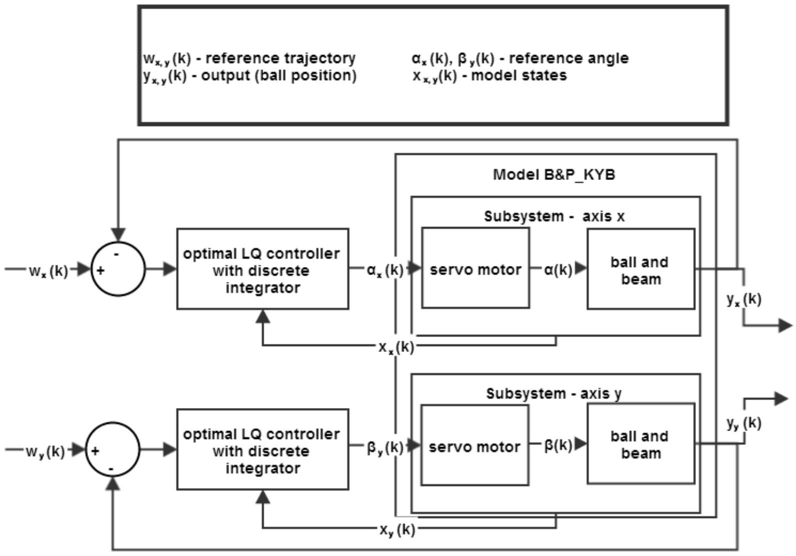
where *w(k)* is reference trajectory and its value in step *k* and K_1, K_2 are matrices calculated from this relation:

$$[-K_1 \quad -K_2] = [-k_x F \quad -I - k_x G]\begin{bmatrix} F - I & G \\ C & 0 \end{bmatrix}^{-1} \tag{20}$$

Base on these steps the control algorithm for simulation and real model of mechatronical system B&P_KYB is being designed. This algorithm is being shown in flowchart in Im. 5. This control algorithm is implemented in scheme Im. 6.



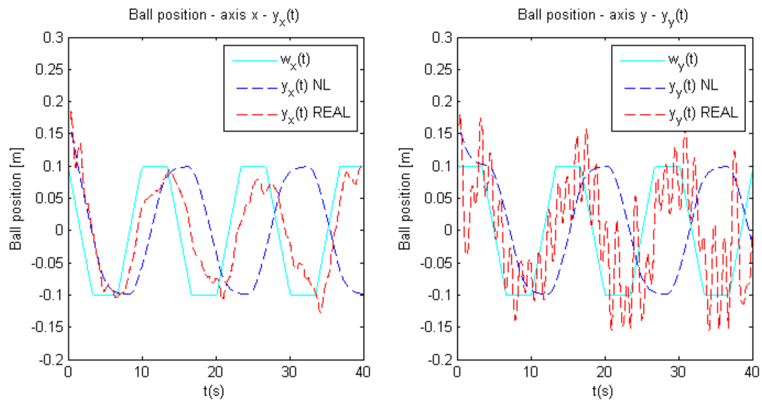
Im. 5 Flowchart of optimal LQ control with an integrator in discrete form



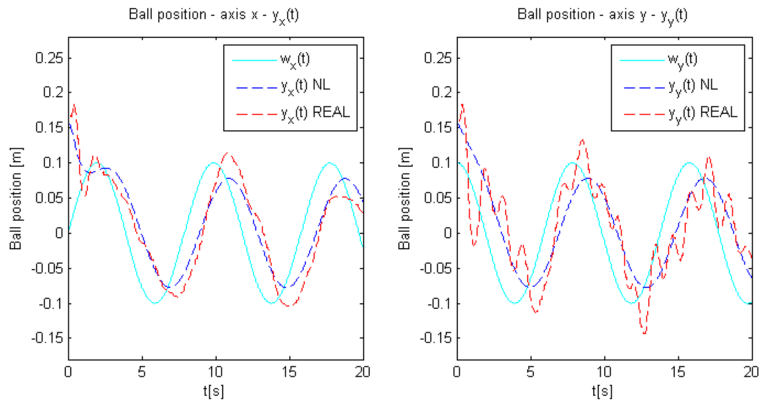
Im. 6 Scheme of optimal LQ control with an integrator in discrete form

C. Verification of design optimal LQ control algorithm with an integrator in discrete form

Algorithm of optimal LQ control with an integrator in discrete form is implemented in the form of newly programmed function *paramLqsC* in *Matlab*. Simulation results of optimal LQ state control for reference trajectory tracking (circle, square) are shown in common graph Im. 7, Im. 8.



Im. 7 Comparison of simulation and real model - square trajectory tracking



Im. 8 Comparison of simulation and real model - square trajectory tracking

IV. Conclusion

This article presents the laboratory model B&P_KYB in terms of structural design, the process of creating simulation model B&P_KYB based on physical laws and design of optimal LQ control of model. Simulation model is created in form Grey box model, which is one of many forms of creating simulation model (in [5] the Black box model form is being presented too). This simulation model after verification is suitable replacement of real model and it is possible to use for simulations. The status description of the model at the selected equilibrium point served for the design of optimal LQ control with an integrator in discrete form of model B&P_KYB for tracking of chosen reference trajectory. Simulation results of optimal LQ control with an integrator in discrete form are not acceptable, because response of real or simulation model B&P_KYB on reference trajectory changing is with delay.

Communication between real model and PC (data acquisition and exchange) with usage of web camera and single-chip microcomputer is very effective. Model using basic, but effective components, enables adaptation of model for actual conditions and requirements. Other models are more complicated, what is limitation for user.

Acknowledgements

This contribution is the result of the Vega project implementation – Dynamic Hybrid Architectures of the Multi-agent Network Control Systems (No. 1/0286/11), supported by the Scientific Grant Agency of Slovak Republic – 40%, and of the KEGA project implementation – CyberLabTrainSystem – Demonstrator and Trainer of Information-Control Systems (No. 021TUKE-4/2012) – 30%, and of the Research and Development Operational Program for project: University Science Park Technicom for innovative applications with knowledge technology support, (ITMS code 26220220182), co-financed by the ERDF – 30%.

References

[1] JADLOVSKÁ, A. - JAJČIŠIN, Š. – LONŠČÁK, R.: Modelling and PID Control Design of Nonlinear Educational Model Ball & Plate. In: Proceedings of the 17th International Conference on Process Control ’09, Štrbské Pleso, High Tatras, Slovakia, 2009, pp. 475–483, ISBN 978-80-227-3081-5. Dostupné na internete: <http://www.kirp.chtf.stuba.sk/pc09/data/abstracts/038.html>
[2] KROKAVEC, D. – FILASOVÁ, A.: Discrete systems. Košice: ELFA, 2006. ISBN 80-8086-028-9.
[3] MODRLÁK, O.: Basics of analysis and synthesis in state space. Materials for study. Liberec: TU – FM, 2004.
[4] NOSKIEVIČ, P.: Modeling and identification of systems. Ostrava: Montanex, 2007. ISBN 80-7225-030-2.
[5] ORAVEC, M.: Control of Mechatronical Systems using .NET Applications. Master thessis. Supervisor: doc. Ing. Anna Jadlovská, PhD. Košice: TU – FEI, 2014.
[6] http://kyb.fei.tuke.sk/laboratoria/miest/V142.php
[7] http://kyb.fei.tuke.sk/laboratoria/modely/gnp.php
[8] http://humusoft.cz/produkty/models/