Predictive Control using the Nonlinear Predictor

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***Abstract*—The main goal of the paper is to introduce the modification of basic predictive control principle with using the nonlinear predictor of controlled system's behaviour. The basic principle and design of predictive control algorithms based on the linear model is mentioned in the paper, too. Testing of introduced control algorithms is carried out by control of the laboratory hydraulic system.**

***Keywords*—hydraulic system, nonlinear dynamical system, nonlinear predictor, predictive control.**

I. Introduction

The paper deals with the predictive control with using a linear and a nonlinear predictor of controlled system's behaviour. Firstly, the basic principle and design of predictive control algorithms based on the linear model are introduced. Next the modification of control algorithms design using the nonlinear predictor is mentioned. As controlled system for algorithms testing the laboratory model of hydraulic system is used, whereby its predictive control based on the linear model was published in [8]. In this article only results obtained by predictive control with nonlinear predictor are presented.

II. Principle of predictive control

The typical feature of the model-based predictive control (MPC) is using the behaviour prediction of controlled physical system in control action computation at each sample. Predicted values of particular quantities are computed on the basis of the controlled system's model.

Regarding to used model of controlled system predictive control algorithms can be divided into two categories:

A. <u>algorithms based on the linear approximation of nonlinear physical system</u> – as discrete transfer function

$$F_s(z^{-1}) = \frac{B_z(z^{-1})}{A_z(z^{-1})} = \frac{\sum_{i=0}^{m} b_i z^{-i}}{\sum_{i=0}^{n} a_i z^{-i}}, \qquad (1)$$

where $B_z(z^{-1})$ is numerator's polynom (order m, coefficients b_i) and $A_z(z^{-1})$ denominator's polynom (order n, coefficients a_i), or in discrete state-space model form

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A_d}\boldsymbol{x}(k) + \boldsymbol{B_d}\boldsymbol{u}(k) \\ \boldsymbol{y}(k) &= \boldsymbol{C}\boldsymbol{x}(k) + \boldsymbol{D}\boldsymbol{u}(k) \end{aligned}, \qquad (2)$$

where $\boldsymbol{A}_d$, $\boldsymbol{B}_d$, $\boldsymbol{C}$ and $\boldsymbol{D}$ are matrices, $\boldsymbol{x}(k)$ is state vector, $\boldsymbol{u}(k)$ is input vector and $\boldsymbol{y}(k)$ is vector of system's output.

B. <u>algorithms, which use the nonlinear model</u> like

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{f}\left(\boldsymbol{x}(t), \boldsymbol{u}(t), t\right) \\ \boldsymbol{y}(t) &= \boldsymbol{g}\left(\boldsymbol{x}(t), \boldsymbol{u}(t), t\right) \end{aligned}, \qquad (3)$$

where $\boldsymbol{x}(t)$ is state vector, $\boldsymbol{u}(t)$ is input vector, $\boldsymbol{y}(t)$ is vector of system's output, $\boldsymbol{f}$ and $\boldsymbol{g}$ are vector nonlinear functions.

In predictive control algorithms, an optimization task is executed for computing the value of control action. Its main principle consists in minimization of criteria function

$$J_{MPC} = \sum_{i=N_1}^{N_p} \boldsymbol{Q}_e\left[\hat{\boldsymbol{y}}(k+i) - \boldsymbol{w}(k+i)\right]^2 + \sum_{i=1}^{N_u} \boldsymbol{R}_u\left[\boldsymbol{u}(k+i-1)\right]^2, \qquad (4)$$

where $\hat{\boldsymbol{y}}(k)$ is vector od predicted output, $\boldsymbol{w}(k)$ is vector of desired value, $\boldsymbol{u}(k)$ is vector of control action values [1]. Values N_1 and N_p represent the prediction horizon and N_u constitutes the control horizon, on which the optimal sequence of control action $\boldsymbol{u}(k)$ is computed, whereby $N_u \le N_p$ [2].

The *Receding Horizon Strategy* is typical for predictive control algorithms [1]. It means the optimal sequence of control action $\boldsymbol{u}_{opt} = \left[\boldsymbol{u}_{opt}(k) \;\; \cdots \;\; \boldsymbol{u}_{opt}(k+N_u)\right]$ is computed on control horizon N_u at each sample k, however only the first element $\boldsymbol{u}_{opt}(k)$ is used as system's input $\boldsymbol{u}(k)$.

The most used approach to predictive control algorithms design can be divided to three steps:

1) the predictor derivation on the basis of the controlled system's linear model,
2) the expression of gradient $\boldsymbol{g}^T$ and Hessian matrix $\boldsymbol{H}$,
3) the optimal sequence of control action computing by criteria function minimization.

We are focused on the MPC algorithms design's particular steps in next paper's part.

III. Design of MPC algorithms with linear predictor

The input of the first step of MPC algorithms design is the linear model of controlled system. The result of this step is the predictor in the matrix form

$$\hat{\boldsymbol{y}} = \boldsymbol{y}_f + \boldsymbol{G}\Delta\boldsymbol{u}, \qquad (5)$$

where $\hat{\boldsymbol{y}}$ is vector of predicted output values, $\boldsymbol{y}_f$ is vector of system's free response and $\boldsymbol{G}\Delta\boldsymbol{u}$ constitutes the system's forced response [1].

The concrete expression of vector $\boldsymbol{y}_f$ and matrix $\boldsymbol{G}$ depends on used form of the physical system's linear model. We are using the state-space model of dynamical systems in this paper, where the control action rate can be written explicitly:

$$\begin{aligned}\boldsymbol{x}(k+1) &= \boldsymbol{A}_d\boldsymbol{x}(k)+\boldsymbol{B}_d\boldsymbol{u}(k-1)+\boldsymbol{B}_d\Delta\boldsymbol{u}(k)\\ \boldsymbol{y}(k) &= \boldsymbol{C}\boldsymbol{x}(k)+\boldsymbol{D}\boldsymbol{u}(k)\\ \boldsymbol{u}(k) &= \boldsymbol{u}(k-1)+\Delta\boldsymbol{u}(k)\end{aligned} \quad . \tag{6}$$

According to [3] it is possible to derive the predictor in form

$$\hat{\boldsymbol{y}} = \underbrace{\boldsymbol{V}\boldsymbol{x}(k)+\boldsymbol{G}_1\boldsymbol{u}(k-1)}_{\boldsymbol{y}_f}+\boldsymbol{G}\Delta\boldsymbol{u}\,, \tag{7}$$

by iterations of state-space model equations (3). In formula (7) the free response $\boldsymbol{y}_f$ is computed on the basis of state quantities current values $\boldsymbol{x}(k)$ and input's previous values $\boldsymbol{u}(k\text{-}1)$, where provided that $\boldsymbol{D}$ is matrix of zeros

$$\boldsymbol{V}=\begin{pmatrix}\boldsymbol{C}\boldsymbol{A}_d\\ \vdots\\ \boldsymbol{C}\boldsymbol{A}_d^{Np}\end{pmatrix},\ \boldsymbol{G}_1=\begin{pmatrix}\boldsymbol{C}\boldsymbol{B}_d\\ \boldsymbol{C}\left(\boldsymbol{A}_d+\boldsymbol{I}\right)\boldsymbol{B}_d\\ \vdots\\ \boldsymbol{C}\left(\boldsymbol{A}_d^{Np-1}+\cdots+\boldsymbol{A}_d+\boldsymbol{I}\right)\boldsymbol{B}_d\end{pmatrix},$$
$$\boldsymbol{G}=\begin{pmatrix}\boldsymbol{C}\boldsymbol{B}_d & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ \boldsymbol{C}\left(\boldsymbol{A}_d^{Np}+\boldsymbol{I}\right)\boldsymbol{B}_d & \boldsymbol{C}\boldsymbol{B}_d & \ddots & \vdots\\ \vdots & \ddots & \ddots & \boldsymbol{0}\\ \boldsymbol{C}\left(\boldsymbol{A}_d^{Np-1}+\cdots+\boldsymbol{A}_d+\boldsymbol{I}\right)\boldsymbol{B}_d & \cdots & \boldsymbol{C}\left(\boldsymbol{A}_d+\boldsymbol{I}\right)\boldsymbol{B}_d & \boldsymbol{C}\boldsymbol{B}_d\end{pmatrix} \quad . \tag{8}$$

It is needed to reduce the matrix $\boldsymbol{G}$ regarding to the length of control horizon N_u

$$\boldsymbol{G}\leftarrow\boldsymbol{G}\begin{pmatrix}\boldsymbol{I}\\ \boldsymbol{0}\end{pmatrix}, \tag{9}$$

where $\boldsymbol{I}$ is unit matrix with dimension $N_u.n_u$, whereby n_u is number of system's inputs[1].

The second step of MPC algorithms design is focused on the work with criteria function's (4) in matrix form – in our case we consider the weighting of control action rate $\Delta\boldsymbol{u}$

$$J_{MPC}=\left(\hat{\boldsymbol{y}}-\boldsymbol{w}\right)^T\boldsymbol{Q}\left(\hat{\boldsymbol{y}}-\boldsymbol{w}\right)+\Delta\boldsymbol{u}^T\boldsymbol{R}\Delta\boldsymbol{u}\,. \tag{10}$$

It should be expressed in suitable quadratic form for computing the optimal sequence of control action rate $\Delta\boldsymbol{u}$. The particular vectors $\hat{\boldsymbol{y}}$, $\boldsymbol{w}$, $\Delta\boldsymbol{u}$, and weighing matrices $\boldsymbol{Q}$, $\boldsymbol{R}$ have their dimensions in accordance with the length of horizons N_p, N_u and numbers of controlled system's outputs and inputs.

After the predictor (7) substitution to the criteria function (10) we can obtain the equation

$$\begin{aligned}J_{MPC} &= \Delta\boldsymbol{u}^T\left(\boldsymbol{G}^T\boldsymbol{Q}\boldsymbol{G}+\boldsymbol{R}\right)\Delta\boldsymbol{u}+\\ &+\left[\left(\boldsymbol{y}_f-\boldsymbol{w}\right)^T\boldsymbol{Q}\boldsymbol{G}\right]\Delta\boldsymbol{u}+\Delta\boldsymbol{u}^T\left[\boldsymbol{G}^T\boldsymbol{Q}\left(\boldsymbol{y}_f-\boldsymbol{w}\right)\right]+c,\end{aligned} \tag{11}$$

which can be expressed as quadratic form

$$J=\Delta\boldsymbol{u}^T\boldsymbol{H}\Delta\boldsymbol{u}+2\boldsymbol{g}^T\Delta\boldsymbol{u}+c\,, \tag{12}$$

where the matrix $\boldsymbol{H}$ and the vector $\boldsymbol{g}^T$ are

$$\begin{aligned}\boldsymbol{H} &= \boldsymbol{G}^T\boldsymbol{Q}\boldsymbol{G}+\boldsymbol{R},\\ \boldsymbol{g}^T &= \left(\boldsymbol{y}_f-\boldsymbol{w}\right)^T\boldsymbol{Q}\boldsymbol{G}.\end{aligned} \tag{13}$$

The minimization of quadratic form (12) is executed in the third step of MPC algorithms design. In opposite to classical dynamical systems control approaches the advantage of MPC algorithms is possibility to respect system's constraints in optimal sequence of control action computing.

We are using the function *quadprog*, which is part of the *Optimization Toolbox* in Matlab for quadratic form (12) minimization. The algorithm for vector of optimal values $\Delta\boldsymbol{u}$ computing by formula

$$\min_{\boldsymbol{u}}\frac{1}{2}\Delta\boldsymbol{u}^T\boldsymbol{H}\Delta\boldsymbol{u}+\boldsymbol{g}^T\Delta\boldsymbol{u}\,,$$
$$\text{with respect to } \boldsymbol{A}_{con}\Delta\boldsymbol{u}\le\boldsymbol{b}_{con}\,, \tag{14}$$

is implemented in *quadprog* function.

The matrix $\boldsymbol{A}_{con}$ and the vector $\boldsymbol{b}_{con}$ should be defined according to system's constraints [2].

While the predictor derivation and Hessian matrix $\boldsymbol{H}$ expression can be executed in advance, the values of gradient $\boldsymbol{g}^T$ and criteria function $\boldsymbol{J}_{MPC}$ minimization are carried out during control loop. This approach to predictive control is the most used all over the world. We were considered it in control of the Hydraulic laboratory model in [8].

IV. Design of MPC algoriths with nonlinear predictor

It is obvious from text before, that the control action value is computed on the basis of the linear model at each sample time in MPC algorithms. Thus, it is very important to approximate the controlled system's dynamics good enough by the linear model. In opposite case the control cannot be adequate, eventually it can destabilize the controlled system.

In this paper's part we present a modified approach to MPC algorithms, where the predictor with nonlinear character is used in computing the control action. As predictor we will use nonlinear differential equations (3) – next only NDE, describing the controlled system's dynamic.

We are engaged in two variants of using the nonlinear predictor in this paper. In the first case, we will use the basic principle of MPC, where we will modify the design procedure in such a case that the system's free response will be computed from the solution of NDE. Any other computations will be based on the predictor in linear form. The second variant is using the fully nonlinear predictor.

Using the nonlinear predictor in free response computing

Regarding to the nonlinear character of model, the computing of system's free response vector values $\boldsymbol{y}_f$ is possible only by numerical methods. For NDE equations (3) we will use solution by 4th ordered Runge-Kutta method [4]. It is necessary to execute the prediction cyclically for N_p samples, where the system's state from sample $k-1$ is used as initial values for computing the solution in current sample k.

In programming way, it is also necessary to modify the function for control action computing in accordance with Fig. 1. In the frame of control action computing, there are only few more numerical computations in control loop in comparison to classical approach with completely linear predictor. Therefore, we suppose only little, possibly neglectable increasing of MPC algorithm's computational time. The advantage of this approach is that the minimization of criteria function, what is the critical part of MPC algorithms regarding to computational time, stays unchanged. It means the optimization problem can be rewritten to quadratic form and computed by the numerical method of quadratic programming. As we have already mentioned it, the nonlinear predictor in system's free response computing constitutes only partial using of predictor's nonlinear character. The nonlinear character affects only values of gradient $\boldsymbol{g}^T$.

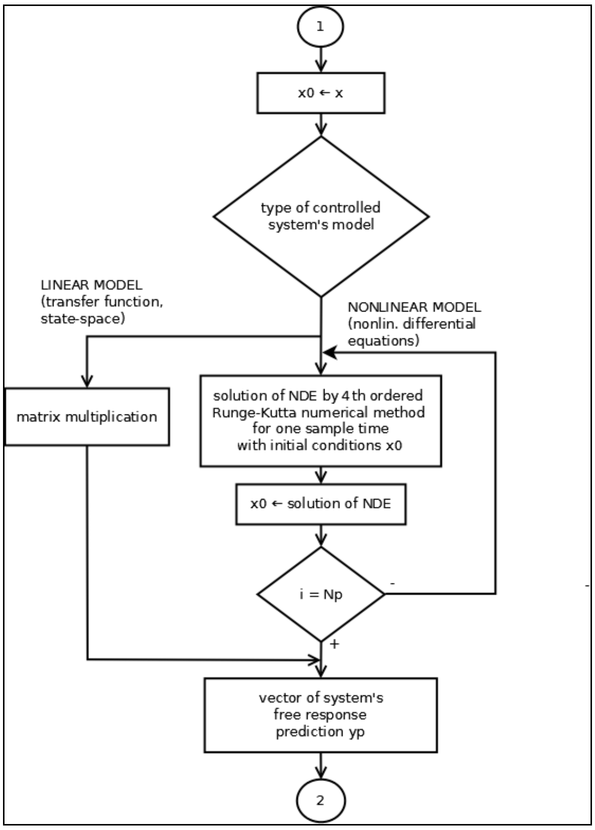

Fig. 1 Part of flow chart diagram for computing the controlled system's free response

In case of systems, which cannot be approximated by the linear model good enough, even this approach cannot be useful to ensure desired behaviour of controlled system. Therefore, in next paper's part we will focused on the nonlinear predictive control, which uses the nonlinear predictor in whole MPC algorithms design.

Nonlinear predictive control

The nonlinear predictive control keeps the basic principle of MPC, however the main idea of this approach is that the model and the predictor are defined by nonlinear functions $\boldsymbol{f}$ a $\boldsymbol{g}$:

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{f}\left[\boldsymbol{u}(k), \boldsymbol{x}(k), \boldsymbol{v}(k), \boldsymbol{z}(k)\right] \\ \boldsymbol{y}(k) &= \boldsymbol{g}\left[\boldsymbol{x}(k)\right] + \xi(k) \end{aligned}, \qquad (15)$$

where $\boldsymbol{u}(k)$ is vector of inputs, $\boldsymbol{x}(k)$ is vector of state quantities, $\boldsymbol{v}(k)$ is vector of measurable disturbances, $\boldsymbol{z}(k)$ is vector of not measurable disturbances, $\xi(k)$ is noise vector and $\boldsymbol{y}(k)$ is vector of system's outputs [5].

It is not possible to use the MPC algorithms design, which was introduced in previous paper's parts in this case, because it is not able to express the predictor in matrix form (7) from equations (15). Thus, it is not possible to rewrite the criteria function in quadratic form (12). For that reason it is necessary to do direct minimization of criteria function by numerical methods for computing the control action value $\boldsymbol{u}(k)$. In our case we used method of *Sequence Quadratic Programming* (SQP). According to [6] it can be proved that solution obtained by SQP is equivalent with Newton-Lagrange solution and it converge quadratically near the minimum point.

We used function *fmincon,* which is part of *Optimization Toolbox* in Matlab for obtaining the nonlinear optimization problem solution. By suitable choice of optimization algorithm (in our case it is SQP) it is possible to find a minimum of arbitrary function *Fun* with respect to constraints defined by $\boldsymbol{A}_{con}$, $\boldsymbol{b}_{con}$ near to the point $\boldsymbol{u}_0$. The simplest syntax of this function in Matlab is

$$\boldsymbol{u} = fmincon(Fun, \boldsymbol{u}_0, \boldsymbol{A}_{con}, \boldsymbol{b}_{con}). \qquad (16)$$

In case of nonlinear predictive control the function *Fun* represents the criteria function (10), where computing of the predicted output values is carried out by numerical Runge-Kutta method again.

Principially, it is possible to express the nonlinear predictive control algorithm by Fig. 2.

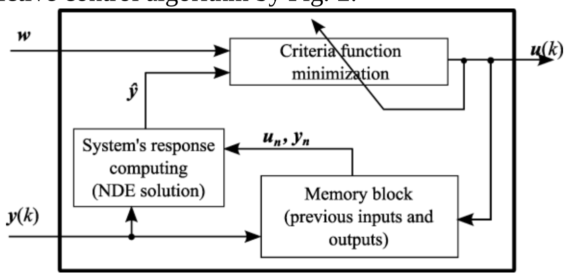

Fig. 2 Nonlinear predictive control algorithm

The minimization of criteria function in nonlinear form spends a lot of computational time, therefore the nonlinear predictive control should be used in control of dynamic systems with slower dynamics, mainly in control of thermal, chemical or hydraulic systems. Especially for that reason we applied the nonlinear predictive control to laboratory model of Hydraulic system, which is located in Laboratory of mechatronic systems at the Department of Cybernetics and Artificial Intelligence (http://kyb.fei.tuke.sk/laben/modely/hyd.php).

V. Hydraulic Laboratory Model Control with MPC Algorithms with Nonlinear Predictor

In this part we are engaged in predictive control of Hydraulic laboratory model, where MPC algorithms with nonlinear predictor, introduced in part IV, are used. Regarding to the fact, that predictive control is primarily used for systems with slow dynamics, we used the hydraulic system. The predictive control of this system based on the linear predictor has already been carried out in [8].

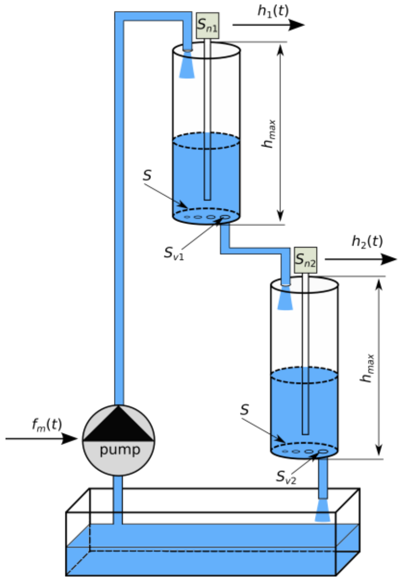

Fig. 3 Hydraulic system of two tanks

The hardware configuration and systemic view of hydraulic system was presented in [7]. Nonlinear differential equations, which represent the model's mathematically-physical description are:

$$\frac{dh_1(t)}{dt} = \frac{1}{S}\left(k_p U(t) - S_{v1}\sqrt{2gh_1(t)}\right)$$
$$\frac{dh_2(t)}{dt} = \frac{1}{S}\left(S_{v1}\sqrt{2gh_1(t)} - S_{v2}\sqrt{2gh_2(t)}\right), \quad (17)$$

where g is the acceleration of gravity and the value k_p expresses the relation between input voltage $U(t)$ and inflow $q_{in1}(t)$. A schematic illustration of hydraulic system is depicted in Fig. 3, whereby particular physical parameters are:

S - intersection of tanks,
S_{v1}, S_{v2} - intersection of outlets of both tanks,
h_{max} - height of tanks (maximal liquid level).

Physical quantities

$f_m(t)$ - pump's motor frequency,
$h_1(t)$, $h_2(t)$ - current levels of liquid in both tanks

constitute system's input and outputs.

Sensors, which scan the current liquid level in both tanks are marked as S_{n1} and S_{n2}.

We present results of Hydraulic system control with predictive control algorithms with nonlinear predictor on next figures. Results are presented as time responses of control action and liquid levels in both of tanks, whereby the goal of control was to ensure desired value of liquid level in the second tank $h_2(k)$. We used algorithms setting in accordance to Tab. 1, where T_s is the sampling time and $\boldsymbol{I}$ is a unit matrix.

T_s	N_p	N_u	$\boldsymbol{Q}$	$\boldsymbol{R}$	constraints
4s	10	2	$1000\boldsymbol{I}$	$0{,}01\boldsymbol{I}$	$u \in \langle 0;8 \rangle V$

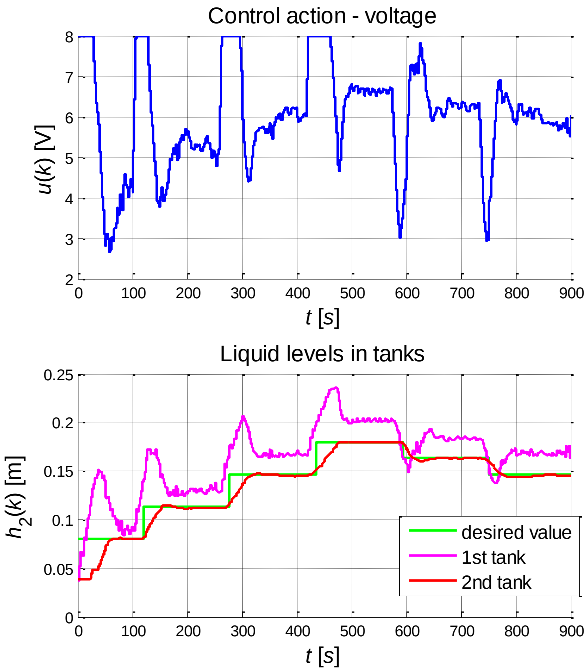

Fig. 4 Time responses of Hydraulic system laboratory model predictive control with nonlinear predictor of system's free response.

By comparison with results published in [8] we can allege that using the nonlinear predictor in dynamical systems predictive control brings better results than with the linear predictor. Especially in control action periodicity and overshooting the desired value by controlled quantity.

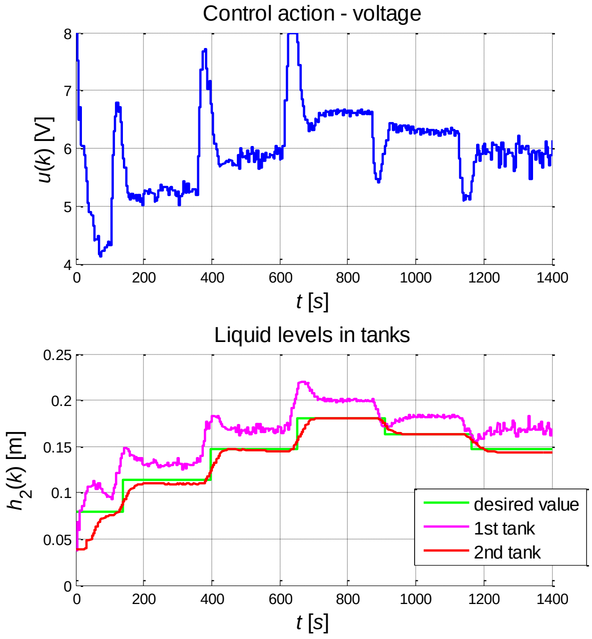

Fig. 5 Time responses of nonlinear predictive control of Hydraulic system laboratory model

VI. Conclusion

We mentioned the basic principle and design of predictive control algorithms with linear predictor. We also introduced two modifications of basic principle with using the nonlinear predictor, which we programmed and tested in laboratory model control. Based on presented results we can consider modified algorithms as suitable for dynamic systems control.

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